This is the first book to discuss current and future applications of waveform diversity and design in subjects such as radar and sonar, communications systems, passive sensing, and many other technologies. Waveform diversity allows researchers and system designers to optimize electromagnetic and acoustic systems for sensing, communications, electronic warfare or combinations thereof. This book enables solutions to problems with how each system performs its own particular function as well as how it is affected by other systems and how those other systems may likewise be affected.

KEY FEATURES
• An excellent standalone introduction to waveform diversity and design.
• Takes a high potential technology area and makes it visible to other researchers, as well as young engineers.
• Documents the beginnings and applications (current and future) of the technology.

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Principles of Waveform Diversity and Design
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Dedication

P. M. Woodward and the Ambiguity Function

Lars Falk

1 Introduction

Fifteen years ago a small book was brought to my attention, *Probability and Information Theory with Applications to Radar* [1]. I had recently returned to radar and wanted to learn more about signal processing. It seemed strange to be directed to a book printed in 1953, but it obviously enjoyed an excellent reputation in view of the number of references found in books and articles.

The slender volume was written in an unfamiliar language but surprisingly easy to read. Finally I realized that this was Bayesian theory expressed in terms of “inverse probability,” the technique used by astronomers in the old days to derive orbital parameters from observational data. The formalism, which simplified the subject and expressions for the accuracy and thresholds of radar measurements, seemed to appear from nowhere. You just had to accept the basic premise that no information should be wasted during processing and interpretation of radar signals. This requirement sounds natural enough, but few authors have dared to use it as a starting point. The principle leads directly to the idea of ideal receivers and matched filters and ultimately to the ambiguity function. The final chapter of Woodward’s book is devoted to waveform design, but the author only mentions in passing that the ambiguity function was introduced for the first time in this book.

Another mystery proved harder to solve. What had happened to the author? After writing the book Woodward seemed to have vanished from the radar scene, or “radar screen” as Professor Nadav Levanon put it. Like other radar experts he wondered what had happened to P.M. Woodward. This question led to a quest that lasted for several years. Now and then I would ask radar scientists what had happened to P.M. Woodward, but without success. Finally, just before giving a talk that was important to me (about crosseye jamming at the AOC conference in Zürich, 2000) I asked a fellow lecturer, Mike Corcoran, the usual question. To my surprise he announced that a Woodward building had recently been inaugurated in Malvern. This did not sound particularly promising, but I soon got a telephone number and started an e-mail conversation. Finally I met Woodward in Malvern in 2003 and could ask him about the book.

2 Three Fields of Excellence

The mystery was easily solved. At the peak of his fame Woodward moved from radars to computers, where he became equally famous. Such moves are unusual, but evidently typical of Woodward. Later he became equally famous in a third field, the construction of mechanical clocks. He seems to have focused on each subject for 20–30 years and achieved outstanding results in all of them.

- Radar, 1941–60
- Computers, 1955–80
- Mechanical clocks, 1975 – present

Philip Woodward was born in 1919 and arrived at the Telecommunication Research Establishment (TRE) at Swanage in 1941. He had studied mathematics for two years at Oxford and found it dull, but TRE was different: “Here was real research with a real purpose.” “The cream of Britain’s scientific talent had been thrown together at TRE under single-minded leadership, and most were under thirty years of age.” [2]
Woodward started by doing numerical calculations on radar antennas affected by reflections from land and sea. This work led to his first paper concerning a clever method of designing radar antennas based on Fourier sampling theory [3]. After the war Woodward remained in Malvern, where he introduced the ambiguity function in 1953. He taught at Harvard in 1956 and then moved on to electronic computers. Woodward became responsible for one of the United Kingdom’s first computers, TREAC, and the first solid state computer, RREAC. In the 1960s his computer software team provided the Royal Radar Establishment with the world’s first implementation of the programming language ALGOL 68.

Woodward’s achievements in horology are equally impressive. He has constructed the best existing mechanical clock, though he modestly assured me that such claims are difficult to verify. His book, *My Own Right Time*, [4] is as highly regarded in the field of horology as *Probability and Information Theory with Applications to Radar* in ours. Woodward contributed dozens of articles to horological periodicals over more than 30 years, including the definitive analysis of balance springs and work on the properties of pendulums. In 2006 the British Horological Institute published a hard-cover collection of 63 articles with his own notes, *Woodward on Time*.

Woodward became a Deputy Chief Scientific Officer at the Royal Signals and Radar Establishment (RSRE). In addition he became Honorary Professor in Electrical Engineering at the University of Birmingham and Visiting Professor in Cybernetics at the University of Reading. In June 2005 the Royal Academy of Engineering gave Woodward its first Lifetime Achievement Award, recognizing him as an outstanding pioneer of radar and for his work in precision mechanical horology. In 2009 he received the IEEE Dennis J. Picard Medal for Radar Technologies and Applications, “For pioneering work of fundamental importance in radar waveform design, including the Woodward Ambiguity Function, the standard tool for waveform and matched filter analysis.”

## 3 Shannon and Woodward

The historical summary of Woodward’s accomplishments in the last section was presented in a banquet lecture at the First International Conference on Waveform Diversity and Design in Edinburgh Scotland in 2004 [5]. The reaction of the audience showed that Woodward’s ambiguity function is a central concept to everyone involved with radar waveform design.

There are several reasons for the success of Woodward’s book [1]. The language is clear and accessible to foreigners, as he mentioned in an e-mail: “One thing that very much pleased me was a letter I had from the translator for the French edition. I no longer have the letter, but he said he had found my style of English easier to translate than that of any other English author with which he had had to contend.” The book looks like a series of lectures, but was actually written in bed. Pleasure is a good starting point, but trouble came later. The preface contains the innocuous phrase: “I have to thank the Chief Scientist, Ministry of Supply, for permission to publish this book.” In fact, permission was only reluctantly granted. The unwillingness may have been motivated, because Soviet authorities immediately translated the book into Russian in 1955.

Woodward wrote his book while radar signal processing was still in its infancy. Yet he managed to deduce almost all modern methods of data processing by applying the general principles suggested by Shannon in his papers on information theory [6,7].

I got a clue to Woodward’s gifts as a scientist when he explained that components in a mechanical clock do not have to be complicated if you design them correctly. Woodward has constructed many tools and the ambiguity function is one of them. One of his scientific heroes is Claude Shannon. They have many talents in common; in particular they have an ability to combine advanced mathematics with engineering that was unusual at the time.

The war moved many scientists into radar and electronic communication. Yet Shannon managed almost single-handedly to transform the subject. I asked Woodward whether he remembered when he first read Shannon’s paper. Yes, he said, in those days there were few journals around and he read
Many people must have felt the same. Wiener and Tuller published similar results concerning the maximum amount of information contained in a signal of known bandwidth and signal-to-noise ratio (SNR). Shannon kindly introduced references to their papers in the book version of his papers [6.7], but this was small consolation. Woodward told me that Tuller described this twist of fate when he visited Malvern several years later. Next day Tuller departed for the USA, but the plane crashed and he was killed, oddly enough near Shannon in Ireland.

Wiener and Tuller made significant contributions, but Shannon completely changed our views on information theory. Woodward recognized this, as shown by the title of his book, which emphasises probability and information theory rather than radar.

At the beginning of our correspondence Woodward noticed that he now had two admirers, a Swedish radar specialist and an English teacher, who thought that his book was the best possible introduction to probability theory for students. This is a reasonable view. The first chapter addresses probability theory without fear of mathematics. The second chapter describes signals and noise and the third explains information theory. After that the reader is ready to study radar theory.

Many radar pioneers thought that radar signals should be described as deterministic functions with added noise. In particular, Gabor wrote influential papers about the amount of information contained in wave fields. He invented holography in the process and got a Nobel Prize for his method of storing and retrieving phase information. Gabor supported Woodward in many ways, but he was apparently too anxious about counting degrees of freedom in terms of time and bandwidth to consider noise and signal amplitudes in the proper way.

The problem is subtle, as shown by the sampling theorem. Most textbooks derive this theorem by considering analytical functions that vanish outside some frequency band or time interval. This method is effective until you consider that in practice most signals are zero both outside a frequency band and a time interval. There are no analytical functions with this property and new methods are required.

Shannon solved the problem by introducing the idea of the probability of a signal defined within an ensemble. The idea of assigning probabilities to signals proved fruitful, but many people thought it was impossible to apply such methods to radar. A transmitted radar signal does not contain information and can not be coded like a message. The received signal contains information about the targets, but coding can only be applied to transmitted signals. Many people thought that this was the end of the argument.

I may inadvertently have shocked Woodward by reminding him of some of the disputes that took place at the first symposium devoted to communication theory and which are duly recorded in the proceedings [8]. Many people failed to understand Shannon’s theory. Others thought that radar signals were fleeting dots on a screen and did not allow much processing. This situation has changed completely; nowadays no one would dream of throwing away data stored in digital memories before all available information has been extracted.

I suggested to Woodward that this was the reason why he abandoned radar. Perhaps he wanted to construct the required tools by developing computers? “There is an atom of truth in that”, he replied in 2003. In September 2008 I gave a talk in Malvern and was delighted to see Philip Woodward in the front row. He had been thinking about my question and wanted to add to his answer.

Woodward told me that at the time he was involved with experiments with a radar system designed to detect submarine snorkels and other objects hidden by sea waves. The idea was to filter in range and Doppler as suggested by Woodward’s theory. Data was stored in a Williams tube, a cathode ray tube that can hold about 1000 bits. The experiment failed for unknown reasons and Woodward decided that processing techniques were not yet mature enough to apply his methods. Computers were already available but computer memories were not fast enough before solid state memories appeared in the 1970s.
4 The Ambiguity Function

Woodward was particularly influenced by Shannon’s paper on communication in noise [7]. He emphasised to me the significance of Shannon’s interpretation of how ambiguities are formed. Similar thoughts can be found in chapter 5 of his book, where the time delay $\tau$ of a single target is analysed. “Shannon (1949) gives an interesting interpretation of the intelligence threshold in communication systems, showing that disconnected ambiguity is liable to occur whenever a signal of low dimensionality is encoded as a signal of higher dimensionality. The simple radar message we have been considering is one-dimensional, namely $\tau$. The signal representing $\tau$, on the other hand, is a waveform of many dimensions.” [1]

Shannon showed how a complicated message can be coded to reduce disturbances. The success of radar during the war was due to its simplicity. There are very few ambiguities in a low pulse-repetition-frequency (PRF) radar in view of the one-to-one correspondence between range and time delay. However, signals with large time-bandwidth products are necessary to improve power management and this inevitably leads to ambiguities. Woodward noticed that in this case the correspondence between time delays and target range will be many to one and investigated the consequences using Shannon’s theory. Shannon and Woodward were both mathematicians with a strong interest in applications. They developed their theories from experience, unlike most scientists who use established techniques to solve accessible problems.

Shannon’s approach led to unexpected concepts like signal entropy and coding. Woodward defined the equally important idea of an ideal receiver, a component that preserves information. It is interesting to notice that Shannon and Woodward knew probability theory but avoided Bayesian terminology, perhaps because statisticians were already waging war on “subjective” methods. Woodward stated his views clearly in the preface [1]: “The present approach was suggested to me by Shannon’s work on communication theory and is based on inverse probability; it is my opinion that of all statistical methods, this one comes closest to expressing intuitive notions in the precise language of mathematics.” Woodward explained this further in an e-mail: “My own knowledge of probability theory had been gained exclusively from Jeffreys, long before Shannon’s work was published. In his book, Jeffreys spends some time defending the “subjective” point of view, and it was only from that defence that I learned of the opposition from statisticians. I knew about insurance companies and life tables, of course, but I regarded statistics as a different subject from probability theory, and still do. To my way of thinking, statistics is concerned with finding the parameters of populations, rather than describing states of mind, whilst probability is concerned with making predictions from hypotheses, no matter how the latter may have been arrived at. It seemed wrong that statisticians should trespass into the domain of probability and inflict their narrow-minded rules on us all. After all, a mathematician can make his own axioms.”

Shannon solved the coding problem by applying knowledge about the statistics of transmitted messages but avoided words like “prior knowledge”. Woodward did not even mention Harold Jeffreys in his book, though Jeffreys’ book on The Theory of Probability is given as a reference in his brilliant paper on radar design [9]. Prior knowledge is obviously useful when one is decoding messages or interpreting radar signals, as Woodward emphasised in chapter 6 of his book: “First, however, a definite assumption has to be made about the prior distribution because ambiguity is a phenomenon which very definitely depends on the extent of prior knowledge.” [1]

Woodward introduced Shannon’s ideas in a manner that is easily understandable to radar engineers. The ideal receiver is defined as a component that preserves information by processing signals in a reversible manner: “Reconstructibility or reversibility, to use the technical term, is the keystone of the subject.” [1]

Woodward and Davies [10,11] used inverse probability to obtain the limits of accuracy of an ideal (correlation) receiver. Their results agreed with those already derived by Marcum in classified papers, but in this case the principle is the important thing. Woodward noticed that phase information omitted in radar measurements contains accurate information about range. This information is in
FIGURE 1  The ambiguity function calculated for a pulse train of seven rectangular pulses (courtesy of Svante Björklund, FOI).

fact used in some modern GPS applications, illustrating the fact that a good theory will often predict new phenomena. This observation is equally true of the last chapter, where the ambiguity function is introduced. Woodward pointed out that a matched filter (correlation receiver) will preserve all information contained in the received signal. To see what happens if the targets are moving, you have to correlate the emitted pulse with a copy shifted in range and Doppler. This leads to the definition of the ambiguity function,

$$\chi(v, \tau) = \int u(t) u^*(t + \tau) \exp(-2\pi i vt) \, dt.$$  

The important point is that the previous analysis can only be improved by adding new information. A typical example is Space-Time Adaptive Processing (STAP), where geometrical information about the possible location of clutter and point jammers is used to suppress interference and locate weak targets.

The ambiguity function has become an indispensable tool for radar signal analysis. Modern computers can present the function in great detail (Figure 1). It is instructive to compare such figures with the contour plots used in Woodward’s book [1].

The conclusion of Woodward’s book offers a fine example of the British art of understatement. Two cases were offered in 1953. Watson and Crick announced the spiral structure of DNA with a carefully formulated final sentence: “It has not escaped our notice that the specific pairing we have postulated immediately suggests a possible copying mechanism for the genetic material.” Woodward formulated an equally subtle conclusion in his book: “The reader may feel some disappointment, not unshared by the writer, that the basic question of what to transmit remains substantially unanswered.” The lack of a unique answer can of course be interpreted more optimistically as a freedom to design waveforms without restricting oneself to the requirements imposed by range and Doppler shifts.

Woodward still preferred the pessimistic note when he summarized the possibilities of removing radar ambiguities in a “futility theorem” published in a 1967 report on radar ambiguity analysis [12]: “Like slums, ambiguity has a way of appearing on one place as fast as it is made to disappear in
another. That it must be conserved is completely accepted, but the thought remains that ambiguity might be segregated in some unwanted part of the $tf$-plane where it will cease to be a practical embarrassment.” This statement recalls Carson’s famous assertion about radio reception: “Noise, like the poor, will always be with us.” The point is that Shannon had refuted Carson’s view in an unexpected way by introducing digital coding. Similarly, Woodward has taught us to look for new possibilities when designing radar waveforms. It is only necessary to take into account the restrictions imposed by nature by expressing them in terms of the ambiguity function.
5 Woodward’s Theorem

A note on naming is indicated here. It is sometimes claimed that Wigner derived the ambiguity function in 1932 within the field of quantum mechanics and that Ville made a similar contribution to signal theory in 1948. This view is anachronistic, because neither Wigner nor Ville was thinking of radar. In fact, Woodward included Ville’s paper among the nine references in his book. The question is what Ville did with his function. Different physical concepts can have the same mathematical form. Stigler stated a law of eponomy: “No scientific law is named after its original discoverer.” This statement applies to Stigler’s Law, but Woodward’s ambiguity function is correctly labeled as long as it used for radar waveform design. It provides a unique description of radar ambiguity while Ville’s function is one of infinitely many functions used to perform time-frequency analysis. The functions look similar but they describe different properties. If the function is used to describe radar, it should be called Woodward’s ambiguity function.

The problem of naming leads to strange situations. A search for Woodward will locate Woodward’s theorem, an elementary approximation for the power spectrum of a frequency modulated (FM) signal derived by Woodward in 1952 [13]:

\[ S(f) \, df \approx \frac{1}{2} A^2 p(f - f_0) \, df. \]

This approximation works well unless the modulation is spike-like [14]. The point is that the approximation is based on probability theory, a subject of particular interest to Woodward. The probability distribution, \( p(f - f_0) \), describes how often a particular frequency is visited by the signal, but it is hardly obvious whether this probability should be regarded as subjective or objective.

6 References

xx Dedications


7 Acknowledgements

The author is indebted to Professor Woodward for offering his views on a number of issues raised in this paper. Professors Nadav Levanon and Hugh Griffiths provided valuable comments on the manuscript. I am indebted to the Royal Academy of Arts and Sciences in Uppsala for travelling grants which made this study possible.
Chaotic Waveform Diversity and Design: Part III—Receiver Synchronization

Taj A. Sturman

1 Introduction

Part I of this chapter on chaotic waveform diversity and design provided the motivation for the use of chaotic waveforms and Part II described techniques for enabling covert communications through chaotic signal generation. Here, in this final part, the practical aspects of receiver synchronization are provided. Receiver synchronization in a covert communication system employing a noise-like digital phase encoding scheme is discussed and a scheme that enables sample timing and symbol synchronization is proposed. Simulation of a carrier-based covert digital communication scheme using differential phase modulation, demonstrating timing synchronization, is reported.

Chaotic signals are potentially useful for secure communications. The phase encoder produces a chaotic signal having an approximately uniform probability density function (PDF) and an impulsive auto-correlation function (ACF); encoder responses are uncorrelated for different initially stored waveforms. The demodulator is shown to recover the data when properly initialized, and recovery from incorrect initialization is accomplished readily (see motivation for chaotic waveforms in Part I and for greater detail refer to [1]).

The phase encoding scheme can, in principle, produce continuous chaotic modulating signals but a digital encoder is envisaged, which produces uncorrelated discrete-time outputs. Such samples are interpolated and a band-limited signal is transmitted. At the receiver, the phase modulation has to be recovered and sampled at appropriate instants to provide the decoder input; the decoder must be correctly synchronized to the phase encoder. System performance was evaluated previously with and without random envelope modulation, assuming perfect phase recovery and perfect synchronization (see assessment in Part II and greater detail can be found in reference [1]). This chapter considers differential phase modulation which avoids the need for carrier phase acquisition and deals with synchronization of sample and symbol timing. Error rate estimates are provided.

2 The Transmitted Signal

The phase of the received signal is to be sampled at, or very close to, the instants corresponding to the original data points, and timing problems are reduced if the phase varies slowly at such points. The proposed system samples the received signal at a high rate and employs digital processing to identify the required timing. The bandwidth of the transmitted signal must therefore be small compared with the sampling rate used in the receiver, and this is accomplished by interpolation before digital-to-analogue conversion, frequency translation and transmission.

Interpolation by upsampling and processing through a finite impulse response (FIR) filter (which has an approximate Nyquist cosine-roll-off impulse response) has been used with an up-sampling ratio, \( L = 16; \) greater detail on waveform synthesis can be found in [2], for example. Appropriate low-pass signals can be produced by using the approximate Nyquist cosine roll-off filter with a roll-off coefficient of 0.9 (the value of the roll-off coefficient is so chosen to produce an impulse response which is significant in the region \( \pm 2\pi \), and insignificant elsewhere so as to reduce the
computational burden of the interpolating filter [3]). Figure 1 shows interpolation of the complex envelope at the output of the phase modulator and Figure 2 shows the PSD of the signal before and after interpolation, ten 2048-point sampled PSDs being averaged.

3 Sample Timing Recovery and Differential Phase Encoding

In the receiver, orthogonal components of the received signal are obtained and linear-phase FIR filters used in each channel to limit the amount of noise. A filter design with 61 coefficients, giving a good compromise between noise suppression and waveform distortion was obtained using the Remez exchange algorithm [4, 5]; its passband is slightly narrower than the transmitted signal spectrum and its stopband attenuation is more than 40 dB.

The receiver is required to sample the received waveform at, or very close to, the original data points. To extract the necessary timing information, a correlation scheme having two correlators, early-middle (EM) and middle-late (ML) are used; three samples are taken in each data symbol interval, one at the sample timing being tested \(t_s\), one at an earlier time \(t_s - \delta\) and one at a later time \(t_s + \delta\). In each case \(N_c\) samples are stored (respectively \(\{x_M\}\), \(\{x_E\}\) and \(\{x_L\}\)) for correlation purposes. The scheme is shown in Figure 3. Two normalized complex correlations are then computed.
from the Early-Middle and Middle-Late correlators, and the differences used to provide an indication of timing error, $I(t_e)$:

$$I(t_e) = |\psi_{EM}(t_e)| - |\psi_{ML}(t_e)|$$

where

$$\psi_{EM} = \frac{\sum_{n=1}^{N} |z_E(n)| z_M^*(n)}{\sqrt{\sum_{n=1}^{N} |z_E(n)|^2 \sum_{n=1}^{N} |z_M(n)|^2}}$$

and

$$\psi_{ML} = \frac{\sum_{n=1}^{N} z_M(n) z_L^*(n)}{\sqrt{\sum_{n=1}^{N} |z_M(n)|^2 \sum_{n=1}^{N} |z_L(n)|^2}}$$

and $z(n)$ are samples of the interpolated complex envelope.

Tests have shown that the optimal value for $\delta$ is $t_e/2$ as this gives the maximum amplitude timing error indicator. Figure 4 plots estimates of $|\psi_{EM}|$, $|\psi_{ML}|$, and $I_e(t)$ for the signal at the output of the receiver filter, without noise.
An algorithm which utilizes \( t_c \) to recover the sample timing, based on a sequential search process has been developed in [1]. Because \( t_c \) contains a zero crossing at zero timing error, the algorithm seeks to locate a zero-crossing, avoiding ambiguity by utilizing knowledge of the gradient.

In the interests of speed, the search is bi-directional, locating the nearest zero-crossing and jumping to the correct sample point if the wrong zero-crossing has been found. In the system simulation, a predominant parameter for sample timing recovery is the correlator size, and here this has been determined by the number of samples which provide a satisfactory timing error indicator even if the received signal-to-noise ratio is poor; experiments suggest that correlation using size \( N_c = 160 \) samples should be adequate.

As demonstrated in [1], in the absence of noise, the maximum time required for locating the correct sampling point is \( (L/4+1)N_c t_s \), which allows for an initially incorrect search direction; simulation has confirmed that this formula provides an estimate applicable when noise is present, assuming an adequate correlation length. Knowledge of this maximum is of significance in the symbol timing recovery process as, during this period of time, the decoder output is erroneous and symbol timing recovery is not possible. An example of sample timing recovery and the waveform at the output of the decoder is provided in [1].

By differentially encoding the phase, the problems of recovering a reference carrier for coherent detection can be avoided, at the expense of degradation in error rate. Such differential encoding may be undertaken before or after interpolation. In the simulation reported here, differential phase encoding precedes interpolation and, in the receiver, sample timing is recovered before differential phase decoding.

For each bit period, \( T_b \), there are \( N_s \) samples, \( (T_b = N_s \delta t) \), and the phase is here differentially encoded using samples separated by \( \delta t \), though other separations are possible. If the output of the encoder is, \( \theta_e[n \delta t] \) the differentially encoded phase \( \theta_{de}[n \delta t] \) at the input to the phase modulator is:

\[
\theta_{de}[n \delta t] = \theta_e[n \delta t] + \theta_e[(n-1) \delta t].
\]  

(2)

The sampled output of the phase modulator is:

\[
s(t) = \sqrt{2P} \cos(2\pi f_c t + \theta_{de}[n \delta t]).
\]  

(3)

At the receiver, the signal would have suffered phase shifting \( \phi(t) \) and the addition of white Gaussian noise \( n(t) \); the received signal is:

\[
r(t) = \sqrt{2P} \cos(2\pi f_c t + \theta_{de}[n \delta t] + \phi(t)) + n(t).
\]  

(4)

The received signal is processed through a parallel pair of orthogonal multipliers with reference carriers \( V_{LOI} \) and \( V_{LOQ} \) having phase \( \phi \) nominally constant and frequency \( f_c \):

\[
V_{LOI} = \sqrt{2} \cos[2\pi f_c t + \phi]; \quad V_{LOQ} = -\sqrt{2} \sin[2\pi f_c t + \phi].
\]  

(5)

The output of each multiplier is then applied to a low-pass filter (LPF), the phase being recovered in the simulation by \( \tan^{-1}[Q/I] \):

\[
\theta_t(n \delta t) = \theta_{de}[n \delta t] + \delta \theta(t)
\]  

(6)

where \( \delta \theta(t) \) is the phase difference between the received signal and the reference carrier,

\[
\delta \theta(t) = \phi(t) - \phi.
\]  

(7)

This phase difference is assumed to be essentially constant over a sample interval, \( \delta t \). The recovered phase is applied to the differential phase decoder whose output \( \theta_{de}[n \delta t] \) is:

\[
\theta_{de}[n \delta t] = \theta_t[n \delta t] - \theta_t[(n-1) \delta t]
\]

\[
= [\theta_{de}[n \delta t] + \delta \theta(t)] - [\theta_{de}[(n-1) \delta t] + \delta \theta(t)]
\]

\[
= \theta_{de}[n \delta t] - \theta_{de}[(n-1) \delta t]
\]

\[
= \theta_e[n \delta t]
\]  

(8)
which can then be applied to the phase decoder for recovery of the binary data waveform. If differential phase encoding is not used, the effect of receiver phase offset is to level-shift the waveform.

### 4 Symbol Timing Recovery

In order to recover the binary data from the noisy binary non-return-to-zero (NRZ) waveform, the output of the phase decoder is processed by an integrator which requires symbol intervals to be correctly timed. Two schemes have been considered for recovering the symbol timing information from the decoder output waveform, one being the early gate-late gate synchronizer scheme and the other, the data transition tracking loop, DTTL ([6,7]).

At the input to the DTTL there is a binary NRZ pulse of duration $T$, with added noise, and this is processed in two parallel integrators, one of which integrates (nominally) over a pulse interval, the other integrating (nominally) over successive half-pulse intervals. The integrators are timed by a timing pulse generator controlled by a digitally filtered error signal obtained, whenever a transition occurs, from the integrator which straddles a transition. Processing in the branch with timing in phase with the data monitors the existence and the polarity of actual transitions of the integrator output data, and the branch with mid-phase timing obtains a measure of the lack of synchronisation. The transition detector examines adjacent matched-filter output samples $a_k - 1$, $a_k$ and produces an output $I_k$ according to the following rules:

- if $\text{sgn}(a_k) = \text{sgn}(a_k - 1)$, then $I_k = 0$
- if $\text{sgn}(a_k) < \text{sgn}(a_k - 1)$ then $I_k = +1$
- if $\text{sgn}(a_k) > \text{sgn}(a_k - 1)$ then $I_k = -1$

Multiplication of the delayed output of the mid-phase integrator by $I_k$ gives the error sample $e_k$, which is applied to a sliding-window averager. Simulation results indicate that accumulating 80 samples provides a good timing error estimate in the presence of noise while not being too large as to allow for reliable tracking of drift between the transmitter and receiver clocks. Integration timing using the DTTL scheme is demonstrated in Figure 5 for zero sample timing error.

In the absence of receiver noise, if the data digits are equi-probable, and statistically independent (with a transition probability of one half), then the average value of the symbol timing error indicator is a linear function of timing error having extreme values which are well defined, their magnitudes
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**FIGURE 6** Demonstrating Symbol Timing Acquisition (Initially $T_e = 0.25T_b$)

being half the maximum output of the mid-phase integrator. The timing error correction used is proportional to the error signal.

In order to prevent erroneous symbol timing error estimates due to incorrect decoder initialization, the number of binary digits corresponding to the maximum acquisition time of the sample timing scheme and the subsequent number of digits corresponding to the decoder impulse response are ignored by the accumulator. An example of the acquisition of symbol timing is demonstrated in Figure 6, which shows integrator timing relative to digit intervals and the variation in estimated error. The initial symbol timing error is $0.25T_b$ and accumulation of the symbol timing error signal starts after the first five digit intervals.

The communication link described earlier has been simulated using the complete data recovery scheme shown in Figure 7. The system employs binary non-return to zero (NRZ) baseband data formatting and random initial encoder waveforms, and samples at the output of the phase decoder are differentially encoded and then applied to the phase modulator the output of which is processed by the transmit filter. BER estimates have been obtained for this scheme using constant envelope and Rayleigh envelope modulation (with data lengths of $10^6$ bits), with the data recovered during timing acquisition being discarded. Error rate estimates with, and without amplitude modulation and including the synchronization scheme described in this paper are presented in Figure 8.

A receiver architecture to enable covert waveform synchronization has been developed and its performance evaluated. The assessment undertaken in this chapter indicates the need for comparatively high levels of SNR for reasonable performance. Methods to enable a reduction to SNR have been proposed in Part II, where also techniques for enhanced covert waveform synthesis have been developed.

Further work in chaotic waveform diversity may include the quantification of trade-offs between communication throughput and signal covertness. In order to support the latter, higher-order techniques of signal detection are likely to be necessary for the purpose of extracting a noise-like chaotic waveform from conventional background noise [9,10].

5 Summary

Effective sample timing and symbol synchronization procedures have been developed heuristically for a particular system which conveys digital information as a noise-like signal [1]. The error rate estimates indicate that employing Rayleigh envelope modulation causes degradation in performance relative to the nominally constant envelope case. These two curves indicate the bounds on performance for this particular phase codec.

It is possible to obtain error rates which lie between these bounds by employing random amplitude modulation with a suitably chosen distribution; there is a trade-off between error rate performance and covert signaling.
FIGURE 7  Receiver scheme
Techniques for enhancing this work have been suggested, whereby the benefits of trading off covert waveform generation to the benefits of increased data capacity are possible; this scheme offers various dimensions to the concept of chaotic waveform diversity.

6 References


7 Acknowledgements

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Radar Signal Analysis and Design Using Frequency Modulation of Chaotic Signals

Ali Ahstari, Gabriel Thomas, Hector Garcés, and Benjamin C. Flores

1 Introduction and Motivation

A transmitting waveform intended for radar applications directly affects the performance of the system. A desirable radar signal provides high resolution, low probability of intercept and interference, optimum use of the available frequency spectrum, high transmitting power, low design cost and generation simplicity. A frequency modulated signal can provide high resolution, high transmitting power and low design cost. Also, a frequency modulated noise signal can provide low probability of intercept and interference and large bandwidth that can potentially yield high resolution. Hence, the use of random noise signals for radar imaging present certain advantages. In particular, the use of chaotic signals as pseudo noise can potentially have advantages for radar imaging because they are also wideband and their generation is easier. A sine wave, frequency modulated by chaotic signals, is an alternative waveform that can yield higher transmitted mean power when peak-power limited transmitters are used.

Figure 1 (a) shows the block diagram of a signal generation scheme that uses digital chaotic sequences as input to a digital frequency modulator. The sampling frequency is chosen based on the...
available frequency range for the operation of the radar. The generated analog signal is shifted to the carrier frequency. Based on the input digital sequence, the spectrum of the FM signal is different (see Fig. 1 (b)).

Unlike random noise FMs, the behavior of chaotic signals after frequency modulation is not fully understood. In this chapter, approaches for analyzing the spectrum of chaotic frequency modulated signals are discussed.

2 Statistical Analysis

We consider one-dimensional discrete time chaotic signals. Specifically, we analyze nonlinear functions $g: \phi \rightarrow \phi$, with domain $\phi \in [-\frac{1}{2}, \frac{1}{2}]$. Let $x(n \Delta t)$ be the discrete version of a chaotic function $x(t)$ such that:

$$x[(n+1)\Delta t] = x(n\Delta t) = g[x(n\Delta t)]$$

where $\Delta t$ represents the sampling interval and $g(\cdot)$ is a nonlinear map. Functions $g$ are chosen so that the sequence of samples $\{x_0, x_1, \ldots, x_n\}$ generated by (1) exhibits fractal behavior. The important property of this sequence is that it is ergodic. Ergodicity in chaotic maps is the result of topological transitivity which is the essential condition for chaos [1]. The immediate objective is to produce a continuous ergodic baseband FM signal with complex envelope $s(t) = A_s \exp(j2\pi K x(t))$ where $A_s$ is the amplitude of the signal, $K$ is its modulation index, and $x(t) = \int_0^t x(\tau) d\tau$. Next, let the discrete version of the signal $s(t)$ be given by

$$s(n) = A_s \exp\left(j2\pi K \sum_{k=0}^{n} x_k\right).$$

The chaotic-based frequency modulated (CBFM) signal $s(n)$ is ergodic and wide sense stationary (WSS) [2].

In this Section, we chose the Bernoulli, Logistic and Tent maps listed in Table 1. The iterated map function can be written as $\phi_{k+1} = F(\phi_k)$. If the initial point of the sequence $\phi_k$ is a random variable $\phi_0$ with probability density function (pdf) $\rho_0$: $\Phi \rightarrow \mathbb{R}^+$, the first iteration of the map $F$ produces a new random variable $\phi_1$ that can be described as $\rho_1 = FPO(\rho_0)$, where $FPO(\cdot)$ is the Frobenius-Perron operator [3]. In general, for the $k^{th}$ iteration: $\rho_k = FPO^k(\rho_0)$, and the FPO describes the time evolution of $\rho_k$ defined as $FPO(\rho(\phi)) = \frac{\partial}{\partial\rho(\phi)} \int \rho(u) du$ [3]. Assuming ergodicity, $\rho_k$ evolves to an invariant pdf $\hat{\rho}$. Keller [4], studied this time evolution and determined that the convergence is bounded according to $\|FPO^k(\rho_0) - \hat{\rho}\| \leq D_c\|\rho_0\|r_{mix}$, where $D_c$ is a constant greater than zero and $r_{mix}$ is the speed of convergence. In fact, $r_{mix}$ expresses the rate of decay of correlations relative to the chaotic sequence $\phi_k$. In other words, lower values of $r_{mix}$ indicate less correlation in the chaotic sequence $\phi_k$. Evidently, the speed of convergence $r_{mix} \in (0,1]$ since $D_c\|\rho_0\|r_{mix} \rightarrow 0$.

### Table 1: One-dimensional chaotic maps and chaotic regime

<table>
<thead>
<tr>
<th>Map</th>
<th>Definition</th>
<th>Chaotic Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bernoulli</td>
<td>$\phi_{k+1} = \begin{cases} B\phi_k + A &amp; \phi_k &lt; 0 \ B\phi_k - A &amp; \phi_k &gt; 0 \end{cases}$</td>
<td>$\phi_k \in [-A, A]$ $1.4 &lt; B &lt; 2$</td>
</tr>
<tr>
<td>Logistic</td>
<td>$\phi_{k+1} = B(A^2 - \phi_k^2) - A$</td>
<td>$\phi_k \in [-A, A]$ $3/2 &lt; AB &lt; 2$</td>
</tr>
<tr>
<td>Tent</td>
<td>$\phi_{k+1} = A - B</td>
<td>\phi_k</td>
</tr>
</tbody>
</table>
Lasota [3] and Hilborn [5] determined analytically the invariant pdf for some one-dimensional chaotic maps. Table 2 summarizes these results.

The FPO provides an analytical tool for studying the evolution (dynamics) of the pdf on an iteration-by-iteration basis. Figure 2 illustrates the dynamics through numerical simulations. In this experiment, the initial sample distribution of each chaotic map has the pdf defined in Table 2. Consequently, the histograms of subsequent samples do not change.

### TABLE 2  Invariant probability density functions

<table>
<thead>
<tr>
<th>Map</th>
<th>Invariant PDF $\hat{\rho}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bernoulli</td>
<td>$1/2A \quad \phi \subset [-A, A]$</td>
</tr>
<tr>
<td>Logistic</td>
<td>$1/\pi \sqrt{(A + \phi)(A - \phi)} \quad \phi \subset [-A, A]$</td>
</tr>
<tr>
<td>Tent</td>
<td>$1/2A \quad \phi \subset [-A, A]$</td>
</tr>
</tbody>
</table>

**FIGURE 2** Histogram time evolution with different initial sample distribution: (a) Bernoulli initial uniform pdf (b) Logistic initial arcsine pdf and (c) Tent initial uniform pdf
3 Sufficient Condition for CBFM Signals to Remain Chaotic

One of the main characteristics of chaotic behavior is the great dependence on initial conditions. A small difference in two initial conditions will give rise to a large trajectory difference after little iteration. The Lyapunov exponent is a number that describes the dynamics of the orbit; it gives a notion of the divergence of nearby trajectories, presenting a method to quantify chaotic behavior.

It is defined as follows: 
\[ \lambda_1 = \frac{1}{n} \ln \left( \frac{\prod_{i=0}^{n-1} g'(x_i)}{\prod_{i=0}^{n-1} g'(x_{i-1})} \right) \]

where \( g'(x) \) is the derivative of \( g(x) \) and \( x \) is the initial condition.

A similar procedure for the \( y \) dimension results in
\[ \frac{d}{dy} s(n) = \left| 1 + \frac{1}{y} g'(h(y, z)) \right| \]

It can be seen that the conclusion obtained for equation (3) cannot be derived from equation (4) because the term \( 1/y \) does not cancel anymore in the calculation of \( E \).

4 Some Chaotic Maps that Satisfy the Sufficient Condition

Seven chaotic maps were chosen for the test. The parameters are chosen so that maps work in their chaotic regime. The first three maps (Logistic, Quadratic and Exponent) do not satisfy (3), while the next four (Tent, Bernoulli, Hopping and Congruent) do. Table 3 shows the first derivative of the chosen maps.

It can be seen from Table 3 that (3) is satisfied by the Tent, Bernoulli, Hopping and Congruent maps. In fact, the first derivative of all of these maps are independent of \( x \) and are also greater
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than 1. Therefore, (3) holds for all values of \( n \). On the other hand, the first derivative of the Logistic, Quadratic and Exponent maps are dependent on \( x \) [7].

5 Spectral Characteristics

In radar range imaging, the resolution of the image is a function of the transmitted signal bandwidth. We already stated that chaotic maps which satisfy (3), produce chaotic CBFM signals. Consequently we expect Tent, Bernoulli, Hopping and Congruent CBFMs to be broadband because chaotic signals are inherently broadband [1]. We cannot predict whether Logistic, Quadratic or Exponent CBFMs are broadband. To address this, the power spectrum of the chaotic signals was calculated. Figures 3 and 4 show the estimated spectrum of the different CBFM signals. It can be seen that the spectra of chaotic CBFMs (Tent, Bernoulli, Hopping and Congruent) are broader in comparison with non-chaotic signals (Logistic, Quadratic and Exponent). This yields better resolution in radar imaging. At this stage, we can conclude that the Tent, Bernoulli, Hopping and Congruent CBFMs are chaotic while others may be chaotic or not.

6 Chaos Reduction

Assuming that a chaotic sequence \( \phi_k \) is ergodic and stationary, we compute an estimate of the power spectral density of (2) by generating the average ensemble autocorrelation of \( M \) signal realizations, and taking its Discrete Fourier Transform (DFT): 

\[
S(f) \approx \text{DFT}\left\{ \frac{1}{M} \sum_{i=1}^{M} R_i(m) \right\}
\]

where \( R_i(m) \) is the ensemble autocorrelation of the \( i \)th realization of (2).

According to Woodward’s Theorem, the spectrum of a random FM signal should have the shape of the pdf associated with the instantaneous frequency \( \phi(t) \):

\[
S(f) \approx \frac{1}{2\Delta f} \rho\left( \frac{f}{\Delta f} \right)
\]  

(5)

FIGURE 3 Power spectrum of CBFM signals which do not satisfy equation (3) in addition to the power spectrum of a Gaussian and uniform noise FM.
where $\hat{\rho}$ is the invariant pdf of $\phi$, and $\Delta f$ is the frequency deviation. Setti [8] proved that for a very small rate of convergence $r_{\text{mix}} \to 0$ the power spectral density of a chaos-based FM signal has a similar behavior as the power spectral density of a random-based FM signal. From the behavior exhibited by the CBFM spectra illustrated in Figure 2, we infer that the Logistic and Tent chaotic maps have a large rate of convergence $r_{\text{mix}}$. The Bernoulli FM appears to fulfill the condition of $r_{\text{mix}} \to 0$. However, the rate of convergence reported in the literature for the three maps is $\frac{1}{2}$.

Setti [8] also describe the “skipping” technique to reduce the rate of convergence. Using the iterated map function $\phi_{k+1} = F(\phi_k)$ to generate the chaotic sequence $\phi_k$, the skipping technique creates a new, one-dimensional chaotic sequence given by

$$y_k = \phi_{\alpha k}$$

where $\alpha$ is a positive integer called the skipping factor. Figure 5 illustrates the new Tent return map $y_k$ generated when we apply the skipping factor $\alpha = 2$.

By reducing the rate of convergence $r_{\text{mix}}$ of a chaotic map, the correlation of the corresponding CBFM signal dies out quickly as can be seen in Figure 6. Also, Figure 7 shows that the Bernoulli FM and Tent FM have white spectra which matches the uniform pdf of the map samples. This figure also shows that the spectrum of the Logistic FM has a shape similar to that of an arc sine pdf.

Table 4 shows the impact of the skipping technique on the average Lyapunov exponent and the chaotic behavior of the FM signals when the skipping technique is applied. Here, the range of confidence is defined as the measured standard deviation of the Lyapunov exponent. Notice that after applying the skipping technique, only the Bernoulli FM remains chaotic.

### 7 Bernoulli Map

Verdin [9] investigated the independence of Bernoulli map samples as a function of the parameter $B$. The figure of merit she considered is the sum of cross terms of the correlation matrix $R$. Accordingly, the samples of a Bernoulli map are approximately uncorrelated for specific values of $B$. Through
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FIGURE 5  Skipping technique, return maps for: (a) Original Tent (b) New sequence $\alpha = 2$

FIGURE 6  Normalized correlation of CBFM signals. Skipping factor $\alpha = 4$: (a) Bernoulli (b) Logistic and (c) Tent
Monte Carlo simulations, Verdin was able to estimate a value of $B = 1.810$ for which the samples of a sequence, become uncorrelated. Figure 8 shows the ensemble autocorrelation of the Bernoulli map for $B = 2$ (invariant uniform case) and $B = 1.81$ (non uniform case). Notice how the autocorrelation dies quickly in the latter case. Alternatively, we can minimize the correlation of adjacent samples

$$ E (\phi_k \phi_{k+1}) = \int_{-A}^{A} \hat{\rho} (\phi) \phi_k \phi_{k+1} d\phi. \quad (7) $$

**FIGURE 7** Spectrum of chaos based frequency modulated signals. Skipping factor $\alpha = 4$: (a) Bernoulli (b) Logistic and (c) Tent

**TABLE 4** Average Lyapunov Exponents

<table>
<thead>
<tr>
<th>Map</th>
<th>$\lambda_{\text{avg}}$</th>
<th>Confidence</th>
<th>$\lambda_{\text{avg}}, \alpha = 2$</th>
<th>Confidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bernoulli</td>
<td>0.2232</td>
<td>$\pm 0.0629$</td>
<td>0.2680</td>
<td>$\pm 0.0956$</td>
</tr>
<tr>
<td>Logistic</td>
<td>0.0021</td>
<td>$\pm 0.0356$</td>
<td>$-0.0031$</td>
<td>$\pm 0.0300$</td>
</tr>
<tr>
<td>Tent</td>
<td>0.0135</td>
<td>$\pm 0.0367$</td>
<td>0.0041</td>
<td>$\pm 0.0306$</td>
</tr>
</tbody>
</table>
We consider the definition of the Bernoulli map equation. Using (7) we obtain
\[ E(\phi_k \phi_{k+1}) = \int_{-A}^{A} B \phi_k^2 \hat{\rho}(\phi) \, d\phi + \int_{0}^{A} A \phi_k \hat{\rho}(\phi) \, d\phi - \int_{-A}^{0} A \phi_k \hat{\rho}(\phi) \, d\phi. \] (8)

We assume a symmetric invariant pdf with zero mean. Setting (8) equal to zero yields
\[ 2A \int_{0}^{A} \phi_k \hat{\rho}(\phi) \, d\phi = B \int_{-A}^{A} \phi_k^2 \hat{\rho}(\phi) \, d\phi \] (9)

We performed a number of Monte Carlo simulations for \( A = 0.5 \) and \( 1.4 < B < 2.0 \), and observed that the histogram of \( \phi \) evolves into an invariant form. However, the distribution was no longer uniform as shown in Figure 9. An empirical expression derived for the invariant pdf is
\[
\hat{\rho}(\phi) = \begin{cases} 
0.727 & \text{for } -0.5 < \phi < -0.315 \\
0.926 & \text{for } -0.315 < \phi < -0.25 \\
1.353 & \text{for } -0.25 < \phi < -0.125 \\
1.088 & \text{for } -0.125 < \phi < 0.125 \\
1.353 & \text{for } 0.125 < \phi < 0.25 \\
0.926 & \text{for } 0.25 < \phi < 0.315 \\
0.727 & \text{for } 0.315 < \phi < 0.5.
\end{cases} \] (10)

Using the experimental pdf (10) in (9) yields an estimate of \( B = 1.62 \pm 0.005 \). Figure 10 shows the correlation \( R(m) \) of the corresponding Bernoulli map. We notice the correlation for a lag \( m = 1 \) approaches zero as expected. However, we also detect that the correlation of samples with a lag \( m = 3 \) exceeds 20%.

In addition, we performed a comparison between the correlation of adjacent samples and the sum of cross correlation terms reported by Verdin [9]. For \( B = 1.81 \), the correlation for \( m = 1 \) is slightly higher that 18%. Moreover, for \( B = 1.5 \) we use the skipping technique described above to verify that the spectrum of the Bernoulli FM signal approaches to the pdf illustrated in Figure 9. In other words, the spectrum has the shape predicted by the Woodward Theorem (5). Figure 11 shows this spectrum.
FIGURE 9  Experimental pdf for Bernoulli map with parameter $B = 1.5$

FIGURE 10  Normalized correlation for a Bernoulli chaotic sequence with $B = 1.62$

FIGURE 11  Bernoulli FM spectrum for $B = 1.5$ and $\alpha = 3$
CHAPTER 59 Frequency Modulation of Chaotic Signals

8 Experimental Results

The technique described here, can be used to easily generate constant envelope wide-band signals. The bandwidth of the signal can easily be controlled by adjusting the conversion rate \( f_s \). Chaotic CBMs and UFM give a flat spectrum in the specified frequency range while non-chaotic CBMs and GFM do not. Therefore, having a specified frequency range for radar imaging, we can easily generate constant envelope wideband signals with optimum use of the available bandwidth for maximum range resolution. To verify the validity of the system in Figure 1, an experiment was conducted. 16,536,000 samples of CBMs, UFM and GFM signals were recorded as audio files with the sampling rate of 11 kHz, using MATLAB. A sound card was used for digital to analog conversion and the spectra were observed using an HP 3580A analog spectrum analyzer. The camera used for obtaining the pictures was a Canon PowerShot A 400. Figure 12 illustrates the spectrum of the GFM, UFM and non-chaotic CBMs.

As expected, UFM has a flat spectrum, while the spectrum of GFM decays at higher frequencies. Logistic CBM, Quadratic CBM, and Exponent CBM also show a decaying spectrum at higher frequencies. These three maps did not satisfy the sufficient condition for chaotic maps to remain chaotic after frequency modulation. Figure 13 illustrates the spectrum of the chaotic CBM signals. It can be seen that also in analog form, the spectrum of the chaotic CBMs and UFM is flatter than non-chaotic CBMs and GFM. All Tent, Bernoulli, Hopping and Congruent maps exhibit a flat spectrum which is in agreement with the sufficient condition provided in the paper. It is interesting to notice that the spectrum of the Bernoulli CBM is even flatter than UFM.

9 Conclusions

This chapter presented a theoretical analysis of CBM signals as well as a sufficient condition for chaotic-based frequency modulated signals to be chaotic. The analysis was based on Lyapunov exponents. The spectra of chaotic CBM signals are flat, while the spectra of non-chaotic CBM signals are non-flat. Consequently, chaotic CBMs have better range resolutions than non-chaotic CBMs. The spectra of the chaotic CBMs are also flatter than those of the GFM (Gaussian frequency
FIGURE 13 Pictures of the spectra using an HP 3580A analog spectrum analyzer. (a) Tent (b) Bernoulli CBFM (c) Hopping CBFM and (d) Congruent CBFM

modulated) signals. Although, the proposed method is not superior to UFM (uniform frequency modulated) signals with respect to the flatness of the spectrum, it is superior to UFM in terms of simplicity and cost.

In radar applications, chaotic signals should not be treated simply as noise signals. This is because changing parameters responsible for producing chaotic signals affects the spectral characteristics of the FM modulated signals. This is unlike noise signals where the spectral characteristics of the FM modulated signal are imposed by the FM modulator and the noise distribution.

These additional degrees of freedom (parameters and maps generating chaotic sequences) result in the advantage of chaos over noise in radar applications. In summary, the proper use of chaotic signals leads to less cost and higher range resolution than noise signals for the application of constant-envelope signals in radar.

10 References

PART III

Imaging Waveforms

Waveform design for imaging through active and distributed sensing is a vibrant area of current radar research. With recent advances in algorithm development and component technologies, radar applications like infrastructure protection, landmine detection, through-the-wall imaging, and identifying improvised explosive devices are emerging as possible affordable sensor technologies for the civil and military sectors. Consequently, designing diverse waveforms with desirable characteristics for this burgeoning field of imaging applications has become a number one priority in many countries.

Until recently, technological constraints have forced traditional radar systems to operate in a relatively rigid manner, with a fixed library of waveforms that can be used for target detection and tracking and, possibly, a set of high-resolution waveforms for target identification. However, current advances in technology allow much greater flexibility in designing imaging waveforms. Since waveform design invariably involves an optimization procedure, such efforts are very mathematical in nature, as evidenced by the chapters in this section.

Although the primary application of this section is the design of waveforms for radar imaging, many of the stated methods are general enough to be applicable across different sensing modalities. For example, the physics-based approach of Varslot et al., where they formulate problems in terms of Green's functions and second-order random fields, is generally applicable to pulse-echo imaging (ultrasound and sonar). The following short vignettes provide brief summaries of current efforts involving imaging waveforms:

- He et al. compare the imaging performance of different frequency selection schemes and show that image quality depends on the scenario (e.g., uniform-random frequency selection achieves better image quality than random frequency selection in proximity to a single target; a uniform-random reduced set of frequencies is superior to random reduction for multiple closely spaced targets).
- Goodman et al. investigate closed-loop strategies for radar waveforms, based on signal-to-noise ratio and mutual information, that adapt their interrogation of the radar channel in response to priorities, prior knowledge, and previous measurements, with the intent of significantly improving performance in difficult and energy-limited environments.
- Garren et al. introduce two alternatives to the standard linear frequency modulated chirp to improve postdetection target discrimination in synthetic aperture radar (SAR) data and show the improvement in the relative signal power for a given target using the two approaches (matched waveform and variable chirp waveform).
- Varslot et al. investigate removing scattering from clutter by modifying (preconditioning) the waveforms that are being transmitted into a scene by determining a filter on the space of transmit waveforms via optimization criteria.
Sparse Stepped-Frequency Waveform Design for Through-the-Wall Radar Imaging

Lin He, Saleem A. Kassam, Fauzia Ahmad, and Moeness G. Amin

1 Introduction

Waveform design for active sensing and imaging is an important area of current research. With recent advances in both algorithm and component technologies, applications such as through-the-wall radar imaging and landmine detection are emerging as affordable sensor technologies in civil and military settings [1-6]. Design of waveforms with desirable characteristics for the broad and growing wideband imaging applications is of great interest and importance [7].

Through-the-wall radar imaging has been recently sought out for surveillance and reconnaissance in urban environments, requiring not only the layout of the building, including types and locations of walls, but also detection and localization of both moving and stationary targets within enclosed structures [1-3]. This technology can also be used by firefighters to detect and locate survivors, by law enforcement officers for enhanced situational awareness and tailored tactical operations, and in search and rescue operations in natural disasters [8,9].

Recent work has provided valuable insights into the effects of array configuration and waveforms on the performance of high resolution radar imaging [10-12]. In this chapter, the design flexibilities for transmit and receive array configurations and transmit signal bandwidth, and their joint role in the ultimate range and angular resolutions of an imaging radar system, are investigated in the context of through-the-wall imaging. We address radar operations using a number of individual elementary transmit waveforms sent from all or selected elements of a transmit array. The transmit waveforms are narrowband CW frequencies (stepped frequencies) spanning a wide transmit band. Stepped-frequency operation has several important advantages when considered for through-the-wall radar imaging [10,13]. It achieves the same effect as that of short duration pulses while avoiding (i) the difficulties associated with the use of short time-duration signals and (ii) some of the complexity of large-bandwidth transmitter and receiver hardware design. Further, the stepped-frequency implementation of a wideband pulse allows changing the emitted power over the signal bandwidth. We can, therefore, compensate for frequency-dependent power attenuation of a wall, thus limiting the signal time-dispersion and preserving the shape and duration of the intended pulse as it travels through the wall [14]. We also note that with stepped-frequency operation the effects of arbitrary transmit waveforms can be synthesized at the receiver. For example, the receive system has the flexibility to reject certain frequency bands where there is heavy interference or where target information is relatively low.

We present approaches based on the “random array” theory and the sum coarray paradigm for the design of sparse stepped-frequency waveforms [12,15-17]. In particular, randomly chosen reduced subset of frequencies and randomly chosen reduced subset of frequencies in uniform subbands, both with and without density tapering, are considered. This amounts to emitting different waveforms from different array antennas, constituting a waveform diversity approach to the through-the-wall radar imaging problem. The primary objective remains, however, the reduction of high data acquisition time in stepped-frequency implementations and to reduce hardware complexity while
maintaining a reasonable image quality. This is the same objective of the new emerging research area of “compressive sensing” [18-20]. In this chapter, we do not apply any optimality to achieve compressed bandwidth or aperture but rather study the effect on image integrity when applying frequency and array thinning strategies.

The chapter is organized as follows. In Section 2, we review the concept of sum coarrays. Section 3 deals with wideband stepped-frequency imaging. Sparse multi-frequency waveform designs using the random and uniform-random frequency selection are detailed in Section 4. Proof-of-concept based on simulated and experimental data is also provided. The performance of the random and uniform-random sparse schemes under multiple target scenario is analyzed in Section 5. Finally, concluding remarks are provided in Section 6.

2 Concept of Sum Coarray

For narrowband far-field active imaging, the sum coarray is defined to be the set [16]

\[ C_S = \{ z : z = x + y, x \in S_T, y \in S_R \} \]  

where \( S_T \) and \( S_R \) are the sets of element position vectors in the transmit and receive apertures, respectively. An example illustrating the sum coarray is shown in Figure 1. The figure shows an active imaging system, which uses a 3-element transmit and a 5-element receive line array for transmission and reception, and its corresponding sum coarray. For this system, the maximum number of points that the sum coarray can have is 15. However, since every pair of transmit and receive elements does not contribute to the formation of a new coarray point, the actual number of points in the sum coarray is 9.

To understand the importance of the sum coarray, let us focus our attention on linear imaging techniques for narrowband imaging of far-field scenes, which are based on the direct exploitation of the Fourier transform relationship between the target reflectivity distribution and the array aperture measurements. In “linear imaging”, the image is the convolution of the point spread function (PSF) with the true reflectivity distribution of the scene [12,16]. The PSF, which determines the basic characteristic of the imaging system, is simply the response of the system to a point source. For weighted beamforming, the PSF is the Fourier transform of a weighting function that has support on the sum coarray. The coarray weighting function is the convolution of the transmit and receive array weightings. In fact, it is only by way of modification of the PSF that the characteristics of the linearly formed image can be influenced. Therefore, the sum coarray provides a characterization of the class of PSFs that an active imaging system can achieve [16].

![Figure 1: Transmit and Receive Line Arrays and the corresponding Sum Coarray](image-url)
The explicit relationship between the PSF \( P(u) \) and the sum coarray weighting \( \gamma(z) \) for far-field narrowband imaging is [16]

\[
P(u) = \int \gamma(z) \exp \left( j 2\pi u \cdot \frac{z}{\lambda} \right) \, dz
\]

(2)

where \( \lambda \) is the wavelength of operation and \( u = (\sin \theta \cos \phi, \sin \theta \sin \phi) \) is the reduced angular coordinate. We note that more precisely, \( P(u) \) is the Fourier transform of the weighting function \( \lambda \gamma(\lambda z) \). Thus, a more explicit definition of the narrowband sum coarray, henceforth referred to as the “physical” sum coarray since it is directly determined by the geometry and locations of the physical transmit and receive arrays, would dilate it by the factor \( 1/\lambda \), i.e., if \( C_S \) is the physical sum coarray of a given array, then the dilated sum coarray, denoted by \( \lambda \cdot C_S \) is

\[
\lambda \cdot C_S = \left\{ \frac{z}{\lambda} : z \in C_S \right\}
\]

(3)

The dilated coarray comes into play when we image the scene using wideband signals. In fact, for wideband operation, the sum coarray is the union of the dilated sum coarrays corresponding to all the frequencies constituting the bandwidth of the transmitted signal [17]. That is,

\[
C_{WB} = \bigcup_{\lambda} \lambda \cdot C_S.
\]

(4)

Note that multi-frequency operation adds extra points at specific locations in the physical sum coarray. This richer sum coarray can be used to enhance the angular resolution achievable with a limited number of array elements [17].

3 Stepped-Frequency Wideband Active Imaging

For wideband imaging, short duration pulses are usually used to achieve good imaging resolution. However, short pulse generation, synchronization, and processing impose strict hardware requirements and thus increase imaging system cost and complexity. Instead of using pulses, a wideband signal can be synthesized using the stepped-frequency approach [10,21]. The stepped-frequency method uses a finite number of monochromatic signals with equi-spaced frequencies to cover the desired bandwidth. The \( K \) discrete frequencies \( \omega_k \) covering the desired bandwidth \( \omega_{K-1} - \omega_0 \) are

\[
\omega_k = \omega_0 + k \Delta \omega, \quad \Delta \omega = \frac{\omega_{K-1} - \omega_0}{K-1}, \quad \text{for } k = 0, 1, \ldots, K - 1.
\]

(5)

Here, \( \Delta \omega \) is the frequency step size.

The stepped-frequency scheme is cost-effective since we do not need expensive equipment for pulse generation and processing. Also, it offers implementation flexibility in the context of waveform design. With just \( K \) transmissions and corresponding receptions of reflections per transmit/receive element pair, it is possible to process the returns and interpret the scene by synthesizing a variety of wideband waveforms using appropriate spectral weighting. These scene interpretations could then be used for relative evaluations among prospective signals for a particular imaging application, such as through-the-wall imaging. However, these advantages come at the expense of data acquisition time. Compared to transmission of a single wideband pulse waveform, the stepped-frequency approach with \( K \) frequencies uses \( K \) longer (narrowband) transmissions. However, if both schemes produce the same total signal energy subject to a maximum amplitude or power constraint, then the pulsed scheme needs to use multiple pulses. It has been shown that the maximum unambiguous range \( R_u \) for stepped-frequency imaging is determined by the frequency step size \( \Delta \omega \) from the relation [21]

\[
R_u = \frac{\pi c}{\Delta \omega}
\]

(6)

where \( c \) is the speed of light. Therefore, for a desired range resolution, i.e., for a given signal bandwidth, the lower limit on the frequency number \( K \) is determined by the farthest range of interest.
CHAPTER 60  Sparse Stepped-Frequency Waveform Design

For wideband far-field active imaging, the PSF of the imaging system employing stepped-frequency signal is given by [10,17]

\[ P(u, R) = \sum_{i=1}^{N_c} \sum_{k=0}^{K-1} \gamma_i S(\omega_k) \exp \left( \frac{j 2\pi}{\lambda_k} (2R - u \cdot z) \right) \]  

(7)

where \( N_c \) is the number of points in the physical sum coarray, \( \gamma_i \) is the weighting applied to the \( i \)th physical coarray location \( z \), \( \lambda_k \) is the wavelength corresponding to the \( k \)th frequency \( \omega_k \), \( S(\cdot) \) is the desired signal spectral weighting, \( R \) is the range, and \( u \) is the reduced angular coordinate. In this case, the estimated target reflectivity distribution \( \tilde{a}(u, R) \) is the two-dimensional convolution of the true target reflectivity distribution \( a(u, R) \) with the PSF \( P(u, R) \) [17]

\[ \tilde{a}(u, R) = a(u, R) \ast P(u, R). \]  

(8)

4  Design of Sparse Stepped-Frequency Waveforms

In this section, we present approaches for the design of sparse stepped-frequency waveforms in order to address the possibly high data acquisition time in stepped-frequency implementations and to further reduce hardware complexity, while maintaining a reasonable image quality.

We consider the idea of a “random array” for narrowband far-field imaging of a scene of radiation sources [15]. A random array has random inter-element spacing, and is one form of aperiodic array. In particular, compared to a periodic array of the same aperture size, a random array allows an average inter-element spacing greater than one-half wavelength, and, therefore, can provide a less costly, thinned or sparse array. Due to the aperiodic element spacing, a thinned random array avoids the aliasing effect (grating lobes) that a periodic (uniformly spaced) sparse array with inter-element spacing greater than one-half wavelength suffers from. However, a thinned random array has the shortcoming of higher sidelobe levels, since it has fewer elements within the same aperture.

Consider a random array of \( N \) elements at locations \( x_n \). The \( x_n \) are chosen from a set of independent random variables with probability density function (pdf), \( p(x) \), which has finite support over some predefined aperture. The PSF of this array is given by [12,15]

\[ P(u) = \frac{1}{N} \sum_{n=1}^{N} \exp \left( -j \frac{2\pi}{\lambda} u \cdot x_n \right). \]  

(9)

The ensemble average of the PSF resulting from many selections of \( N \)-element locations is given by

\[ \mathbb{E}[P(u)] = \frac{1}{N} \sum_{n=1}^{N} \exp \left( -j \frac{2\pi}{\lambda} u \cdot x_n \right) \]  

(10)

where \( \mathbb{E} \) denotes the mean of the random variable \( x \). Since the many selections of \( x_n \) come from the same pdf, the mean of \( \exp (-j \frac{2\pi}{\lambda} u \cdot x_n) \) is independent of \( n \). Therefore,

\[
\mathbb{E}[P(u)] = \exp \left( -j \frac{2\pi}{\lambda} u \cdot x \right) \left( \frac{1}{N} \sum_{n=1}^{N} 1 \right) \\
= \exp \left( -j \frac{2\pi}{\lambda} u \cdot x \right) \\
= \int p(x) \exp \left( -j \frac{2\pi}{\lambda} u \cdot x \right) dx
\]  

(11)
We observe from equation (11) that the ensemble average of \( P(u) \) is the same as the desired PSF \( P_0(u) \) of a periodic array of the same aperture size provided that the pdf \( p(x) \) of the random array element location is chosen to emulate the designed array weighting function \( w_0(x) \), i.e., \( p(x) = w_0(x) \). This implies that \( P(u) = P_0(u) \). The random array can, therefore, be thinned without altering the expected pattern.

On the other hand, for a random array with PSF \( P(u) \) as given by equation (9), the ensemble average power pattern is given by [12,15]

\[
|P(u)|^2 = P(u)P^*(u) = \frac{1}{N^2} \left( \sum_{n=1}^{N} 1 + \sum_{n=1}^{N} \sum_{i=1}^{N} \exp \left( -j \frac{2\pi}{\lambda} u \cdot x_i \right) \exp \left( j \frac{2\pi}{\lambda} u \cdot x \right) \right)
\]

\[= \frac{1}{N^2} \left( N + \exp \left( -j \frac{2\pi}{\lambda} u \cdot x \right) \exp \left( j \frac{2\pi}{\lambda} u \cdot x \right) (N^2 - N) \right)
\]

\[= \frac{1}{N} + |P(u)|^2 \left( 1 - \frac{1}{N} \right).
\]

The first term on the right in equation (12) is an additive, angle-independent factor of strength \( 1/N \), which implies that the control of the sidelobe region of the average power pattern of a random array is limited. Clearly, it is the number of sensors and not their locations that determines the sidelobe power level. Therefore, although a random array can achieve a desired expected PSF by proper choice of the pdf \( p(x) \), its average power pattern must suffer. Equation (12) shows that for \( p(x) = w_0(x) \), the shape of the main lobe and nearby sidelobe region where sidelobes have average power levels much greater than \( 1/N \) is essentially the same as that of \( |P_0(u)|^2 \). Away from the main lobe where the sidelobe power level of the desired pattern is less than \( 1/N \), the character of the desired power pattern is overwhelmed by the random component.

Now consider stepped-frequency wideband active imaging. Recall that the wideband coarray is the union of all the dilated coarrays corresponding to different frequencies in the spectrum of the transmitted signal. If we randomly drop some frequencies from the full set of K frequencies, the resulting wideband coarray points have almost the same distribution as that of the wideband coarray points corresponding to the full frequency set. This is illustrated in Figure 2. Figure 2(a) shows the wideband coarray, corresponding to a 29-by-15 element 2D monostatic array with an inter-element spacing of 44.45 mm on a square grid, for the full 201 component stepped-frequency signal covering 2 to 3 GHz frequency band. The wideband coarray for 25% randomly retained frequencies is depicted in Figure 2(b). This random selection of frequencies is analogous to the selection of element locations for randomly thinned, sparse arrays with an element count less than that of a periodic array having the same aperture size. Exploiting this analogy and from equations (7), (11), and (12), we conclude that the resulting sparse-in-frequency array has a thinned wideband coarray, which achieves a desired PSF on average, but with larger sidelobe levels. This thinned-in-frequency wideband array reduces the data acquisition time considerably compared to a full stepped-frequency system.

In applying the idea of random frequency selection, if we use an appropriate random frequency selection scheme, we can potentially improve the image quality through some additional control over the sidelobe levels, at least near the main lobe. One such scheme is the "uniform-random" frequency selection strategy that we describe next. Starting with a full set of stepped frequencies spaced by \( \Delta\omega \), we divide it into a number of frequency groups. Each group consists of the same number of contiguous frequencies. Then, in each group, we retain some fixed smaller number of randomly chosen frequencies to compose the multi-frequency waveform. For example, consider a 100-component stepped-frequency set, and assume that we want to use only 20% of the frequencies. We can divide the 100 frequencies into 20 frequency groups and randomly select one out of the five frequencies in each group. Compared to random frequency selection from the full set, this
uniform-random frequency selection makes the selected frequency distribution more uniform for the limited number of components.

A simple analysis can be given for the PSF in the uniform-random multi-frequency approach, for the far-field case. Suppose we “uniform-randomly” choose $M$ frequencies from the full stepped-frequency set for each physical coarray point; for the $i$th physical coarray point, these can be expressed as

$$\omega_{mi} = \omega_{m1} + \Delta \omega_{mi}, \quad m = 1, 2, \ldots, M$$  (13)
where \( \omega_{m1} \) is the first frequency in the \( m \)th frequency group, and \( \Delta \omega_{m1} \) is the frequency difference between the randomly selected frequency \( \omega_{m1} \) and frequency \( \omega_{m1} \) in the \( m \)th frequency group for the \( i \)th physical coarray point. Assuming all frequencies have unit complex amplitude and using equation (7), the PSF for the uniform-random multi-frequency approach can be expressed as

\[
P_{\text{uniform}}(\textbf{u}, R) = \sum_{i=1}^{N_c} \sum_{m=1}^{M} y_i \exp \left( j \frac{\omega_{m1} + \Delta \omega_{m1}}{c} (2R - \textbf{u} \cdot \textbf{z}) \right). \tag{14}
\]

Note that if \( \max(|\Delta \omega_{m1} (2R - \textbf{u} \cdot \textbf{z})/c|) \) is sufficiently smaller than 1 radian, equation (14) can be approximated as

\[
P_{\text{uniform}}(\textbf{u}, R) \approx \sum_{i=1}^{N_c} \sum_{m=1}^{M} y_i \exp \left( j \frac{\omega_{m1}}{c} (2R - \textbf{u} \cdot \textbf{z}) \right). \tag{15}
\]

If the frequency group step size \( \omega_{m1} - \omega_{m-1} \) is small enough, the PSF expressed in equation (7) is approximately proportional to the PSF of equation (15). This indicates that for the case where

\[
\max(|\Delta \omega_{m1} (2R - \textbf{u} \cdot \textbf{z})/c|) \leq 1/2, \tag{16}
\]

the PSF using the uniform-random multi-frequency approach has almost the same shape as that of the PSF using the full stepped-frequency set. We note that for the sidelobes near the main lobe where \( R \) and \( \textbf{u} \) are small, the above condition is satisfied. Therefore, compared to the random frequency approach, the uniform-random frequency selection can be expected to improve image quality at least around the target location, while keeping its advantage of lower data acquisition time. Although the analysis above is for the far-field case, simulations show that the result gives a good characterization of the image quality for near-field imaging.

For illustration, consider a two-dimensional array of transceivers, shown in Figure 3(a), located in the \( x-y \) plane at \( z = 0 \). The length of the array is 1.2446 m along \( x \) and 0.6223 m along \( y \), with an inter-element spacing of 44.45 mm on a square grid. We use a stepped-frequency signal of 1 GHz bandwidth centered at 2.5 GHz, with 201 steps of size 5 MHz. Subarray aperture syntheses was assumed for the data collection [10], wherein the 201 CW frequencies are transmitted from a single array element and the returns are received at the same array location only, which indicates a monostatic data collection radar operation. This process is then repeated for the next transceiver location until all of the 29-by-15 array locations are exhausted. Note that, because of monostatic operation, the corresponding physical sum coarray is twice the size of the array, but has the same number of points as the number of array elements [See Fig. 3(b)].

We first compare the simulated images of a single point target, located along the positive \( z \)-axis with \( (x, y, z) \) coordinates \((0, 0, 3.5)\) m, for the above synthetic array using three different discrete frequency sets covering the 1 GHz bandwidth, as explained below. These frequency sets can be viewed as different transmit waveforms, and can be cast as a “waveform diversity” approach. In Case I, we use all 201 frequencies for each array location. In Case II, we use 50 randomly chosen frequencies from amongst the 201 frequencies for each array location (approximately 25% of the full set). In Case III, we choose 50 frequencies in a “uniform-random” manner for each array location; specifically, we randomly choose one frequency out of each group of four contiguous frequencies for the entire 201 frequencies. Figures 4(a) and 4(b), respectively, show the frequency sets for the top left and the center array elements. Note that because of monostatic operation, each coarray point has contribution from a single distinct transceiver location only and, therefore, the same retained set of frequencies as the corresponding array element. Figure 5 shows the B-scan (horizontal cut through the 3D volume) images with \( y = 0 \) for the three multi-frequency schemes. The plots in the left column of Figure 5 zoom-in on the area around the target location, while those in the right column present a larger extent of the imaged scene. Compared to the full-frequency array (Case I), the sparse multi-frequency arrays (Case II and III) require substantially less data acquisition time and processing time to generate one image. However, there is some sacrifice of image quality.
observe that the sparse multi-frequency schemes display higher background sidelobe levels, but the imaging resolution remains almost the same as that for Case I. The controlled uniform-random choice of frequencies in Case III achieves similar performance around the target location as that obtained using the full stepped-frequency set. However, the sidelobe clutter far from the main lobe in the image using uniform-random frequency set is more like that using the random frequency set, due to the random components. We note that the uniform random multi-frequency approach (Case III) keeps the efficiency of Case II, requiring a quarter of the data acquisition time of the full-set (Case I).

Next, we compare the performance of the sparse multi-frequency schemes using real data. A wideband synthetic aperture radar imaging system was set up in the Radar Imaging Lab at Villanova University. A stepped-frequency signal with 201 steps covering the 2 to 3 GHz frequency band was used for imaging. A conducting sphere of 10 in diameter, shown in Figure 6, located at \((x, y, z)\) coordinates \((0.28, 0.107, 2.73)\) m, was used as the target \((y = 0\) corresponds to a height of 1.22 m above the ground). The 29-by-15 element array of Figure 3(a) was synthesized by moving a quad-ridge horn antenna, mounted on a Field Probe Scanner, to different locations forming the array aperture. The data measurement process for each transceiver location consisted of the following steps.

1. **Background measurement.** This is a measurement made of the received complex amplitude of all 201 monochromatic signals. The purpose of this experiment is to measure the clutter characteristics of the lab, with only one exception: the 10 inch sphere target is not present.
2. Target measurement. This is a complex amplitude measurement of the received signal across all 201 frequencies using the same exact setting as in the first experiment, but including the 10 in sphere.

3. Background subtraction. The background data set is subtracted from the target data set for clutter reduction, although clutter due to mutual interaction between the sphere and the lab remains.
FIGURE 5  B-scan single target images ($y = 0$ m) using (a) full frequency set, (b) random frequency set, (c) uniform-random frequency set
This final data set, after clutter subtraction, was used for processing. Figure 7 shows the B-scan images, corresponding to \( y = 0.107 \text{ m} \) (height of center of the target), using the same frequency sets as those described for Cases I, II, and III above. We observe that the uniform-random multi-frequency approach (Case III) provides better image quality around the target compared to the random frequency set (Case II). Note that in this case, the data acquisition time consists of not only the time it takes to sweep across all frequencies, but also the time required for physically moving the antenna to the various locations constituting the array aperture. In fact, the latter is the dominant component of the total data acquisition time. Thus, in this case, the use of reduced frequency sets, which retain only 25% of the full frequency set, does not translate into a 25% reduction in the data acquisition time. In contrast, the alternate implementation of the aperture synthesis scheme, which was assumed in the simulation example of Figure 5 above, where all the elements of the intended array are physically present and share a single processing channel via a multiplexer, provides a one-to-one correspondence between the smaller number of frequencies and reduction in data acquisition time.

5 Performance under Multiple Target Scenario

In a real environment, in addition to the “main” target of interest, there may be strong secondary targets present in the scene. The main and secondary targets may form a close cluster or the secondary targets may be present away from the main isolated target. For the former case, where all targets lie within some limited region of the main target plane, the performance of both random and uniform-random multi-frequency approaches is expected to be similar to the single target scenario. This can be attributed to the good image quality around the main lobe and the first few sidelobes using the sparse stepped-frequency schemes. For the latter case where the main target is isolated from the strong secondary targets, the uniform-random scheme may not show any advantage compared
FIGURE 7  B-scan images (at target height) obtained using experimental data and (a) full frequency set, (b) random frequency set, (c) uniform-random frequency set
FIGURE 8  B-scan images of a close cluster of targets \((y = 0 \text{ m})\) using (a) full frequency set, (b) random frequency set, (c) uniform-random frequency set.
to the random scheme. The reason is that the sidelobe clutter far from the mainlobe, arising from these strong secondary targets, may dominate the sidelobe background level near the isolated target of interest. To improve the image quality in this case, we consider a “tapered” sparse frequency selection. In the “tapered” frequency selection, we select more frequencies in the middle of the frequency band, and fewer towards the ends of the frequency band. With properly tapered frequency selection, we can expect the levels of sidelobes away from the main lobe to be lower as compared to those of random and uniform-random frequency selection. The price we pay for this improvement is some degradation in range and angular resolutions. Therefore, we can attempt to balance image resolution and sidelobe clutter levels by proper tapered frequency selection.

For illustration, we first consider a scene consisting of two targets having the same reflectivities, located at \((0, 0, 3.5)\) and \((0.5, 0, 2.6)\), the units being meters. Figure 8 shows the corresponding B-scan images with \(y = 0\) using the full, 25% random, and 25% uniform-random frequency sets. The array and signal parameters are the same as that described in Section 4. As expected, both the random and uniform-random schemes allow a good tradeoff between complexity and data acquisition time.

Next, we consider a scene of five targets having the same reflectivities, located at \((0, 0, 3.5)\), \((3.2, 4.3, 9.6)\), \((-4.8, 5.5, 0.6)\), \((3.5, -4.8, 8.5)\), and \((-3.3, 5.9, 7.9)\), the units being meters. The “main” target at \((0, 0, 3.5)\) m is far away from the other four targets. For this scene, in addition to the full (Case I), 25% random (Case II), and 25% uniform-random (Case III) frequency sets, we use two tapered frequency selection sets as well. In Case IV, we separate the 201 frequencies into 5 groups, each group having 40 contiguous frequencies. We randomly choose \(\{5, 10, 20, 10, 5\}\) frequencies in the 5 groups, to obtain respectively \(\{12.5\%, 25\%, 50\%, 25\%, 12.5\%\}\) groups of “random-tapered” frequencies adding up to 25% of the 201 frequencies. In Case V, we uniformly-randomly select \(\{5, 10, 20, 10, 5\}\) frequencies within the 5 groups to obtain groups of \(\{12.5\%, 25\%, 50\%, 25\%, 12.5\%\}\) “uniform-tapered” frequencies. Figure 9 shows the B-scan images with \(y = 0\) using the un-tapered multi-frequency schemes. As expected, the advantage of the uniform-random (Case III) over the random (Case II) scheme is diminished. The images corresponding to the tapered schemes are illustrated in Figure 10. Compared to the un-tapered sparse multi-frequency schemes (Cases II and III) which have almost the same imaging resolution as that for Case I, the tapered sparse multi-frequency schemes (Cases IV and V) display lower imaging resolution. Moreover, the tapered schemes have lower background sidelobe levels compared to their un-tapered counterparts, more so for the uniform-random case than the random schemes. More specifically, the average background sidelobe level for the tapered uniform-random scheme is 1.5 dB lower than that of its un-tapered counterpart, whereas the average background sidelobe levels for the tapered and un-tapered random schemes differ by 0.5 dB.

6 Conclusions

While the stepped-frequency waveform synthesis for wideband imaging leads to a flexible and simplified implementation of the transmit and receive beamformer, the sparse multi-frequency array with proper frequency selection can obtain reasonable image quality with the same advantages and much less data acquisition time. Random, uniform-random, tapered random, and tapered uniform-random reduced frequency selection schemes have been presented for indoor imaging. It is shown that for a single target scene, the uniform-random frequency selection can achieve better image quality, compared to the random frequency selection method, around the target location. For a scene in which multiple targets are close together, uniform-random reduced set of frequencies retains the advantage over random reduction. However, in the presence of other strong targets away from main target cluster, little advantage of uniform-random over random remains. In such situations, tapered density of retained frequencies is shown to provide improved image quality.
FIGURE 9 B-scan multiple target images ($y = 0$ m) using (a) full frequency set, (b) random frequency set, (c) uniform-random frequency set.
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FIGURE 10 B-scan multiple target images \( y = 0 \) m using (a) random-tapered frequency set, (b) uniform-tapered frequency set

7 References


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This is the first book to discuss current and future applications of waveform diversity and design in subjects such as radar and sonar, communications systems, passive sensing, and many other technologies. Waveform diversity allows researchers and system designers to optimize electromagnetic and acoustic systems for sensing, communications, electronic warfare or combinations thereof. This book enables solutions to problems with how each system performs its own particular function as well as how it is affected by other systems and how those other systems may likewise be affected.

KEY FEATURES
• An excellent standalone introduction to waveform diversity and design.
• Takes a high potential technology area and makes it visible to other researchers, as well as young engineers.
• Documents the beginnings and applications (current and future) of the technology.

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