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CHAPTER

2

The Radar Range Equation

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Chapter Outline

2.1 INTRODUCTION

As introduced in Chapter 1, the three fundamental functions of radar systems are to search for targets, to find targets, and in some cases to develop an image of the target. In all of these functions the radar performance is influenced by the strength of the signal coming into the radar receiver from the target of interest and by the strength of the signals that interfere with the target signal. In the special case of receiver thermal noise being the interfering signal, the ratio is called the signal-to-noise ratio (SNR), and if the interference is from a clutter signal, then the ratio is called signal-to-clutter ratio (SCR). The ratio of the target signal to the total interfering signal is the signal-to-interference ratio (SIR). A signal is never composed of target alone. There is always some noise in addition to the target signal. The radar performance depends on the target-plus-noise to noise ratio.

In the search mode, the radar system is programmed to reposition the antenna beam in a given sequence to "look" at each possible position in space for a target. If the signal-plus-noise at any spatial position exceeds the interference by sufficient margin, then a *detection* is made, and a target is deemed to be at that position. In this sense, *detection* is a process by which, for every possible position for a target, the signal (plus noise) is compared with some threshold level to determine if the signal is large enough to be deemed a target of interest. The probability that a target will be *detected* is dependent on the probability density function (PDF) of the interfering signals, the SIR, the target fluctuation characteristics, and the threshold level to which the signal is compared, which depends on the desired probability of false alarm, P_{FA}. The detection process is discussed in more detail in Chapters 3 and 15, and special processing techniques designed to perform the detection process automatically are discussed in Chapter 16.

In the tracking mode, the accuracy or precision with which a target is tracked also depends on the SIR. The higher the SIR, the more accurate and precise the track will be. Chapter 19 describes the tracking process and the relationship between tracking precision and the SIR.

In the imaging mode, the SIR determines the fidelity of the image. It determines the dynamic range of the image—the ratio between the "brightest" spots and the dimmest on the target. The *SIR* also determines to what extent false scatterers are seen in the target image.

The tool the radar system designer or analyst uses to compute the SIR is the *radar range equation* (RRE). A relatively simple formula, or a family of formulas, predicts the received power of the radar's radio waves "reflected"[1] from a target and the interfering noise power level and, when these are combined, the SNR. In addition, it can be used to calculate the power received from surface and volumetric clutter, which, depending on the radar application, can be considered to be a target or an interfering signal. When the system application calls for detection of the clutter, the clutter signal becomes the target. When the clutter signal is deemed to be an interfering signal, then the SIR is determined by dividing the target signal by the clutter signal. Intentional or unintentional signals from a source of electromagnetic (EM) energy remote from the radar can also constitute an interfering signal. A noise jammer, for example, will introduce noise into the radar receiver through the antenna. The resulting SNR is the target signal power divided by the sum of the noise contributions, including receiver thermal noise and jammer noise. If the jammer is a false target jammer, then the SIR is found by dividing the target signal received by the jammer power received. Communications signals and other sources of EM energy can also interfere with the signal. These remotely generated sources of EM energy are analyzed using one-way analysis of the propagating signal. The one-way link equation can determine the received signal resulting from a jammer, a beacon transponder, or a communications system.

This chapter includes a discussion of several forms of the radar range equation, including those most often used in predicting radar performance. It begins with forecasting

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[1]Chapter 6 shows that the signal illuminating a target induces currents on the target and that the target reradiates these electromagnetic fields, some of which are directed toward the illuminating source. For simplicity, this process is often termed *reflection*.

the power density at a distance R and extends to the two-way case for monostatic radar for targets, surface clutter, and volumetric clutter. Then radar receiver thermal noise power is determined, providing the SNR. Equivalent but specialized forms of the RRE are developed for a search radar and then for a tracking radar. Initially, an idealized approach is presented, limiting the introduction of terms to the ideal radar parameters. After the basic RRE is derived, nonideal effects are introduced. Specifically, the component, propagation, and signal processing losses are introduced, providing a more realistic value for the received target signal power.

2.2 POWER DENSITY AT A DISTANCE *R*

Although the radar range equation is not formally derived here from first principles, it is informative to develop the equation in several steps. The total peak power (watts) developed by the radar transmitter, P_t, is applied to the antenna system. If the antenna had an isotropic or omnidirectional radiation pattern, the power density Q_i (watts per square meter) at a distance R (meters) from the radiating antenna would be the total power divided by the surface area of a sphere of radius R,

$$Q_i = \frac{P_t}{4\pi R^2} \tag{2.1}$$

as depicted in Figure 2-1.

Essentially all radar systems use an antenna that has a directional beam pattern rather than an isotropic beam pattern. This means that the transmitted power is concentrated into a finite angular extent, usually having a width of several degrees in both the azimuthal and elevation planes. In this case, the power density at the center of the antenna beam pattern is higher than that from an isotropic antenna, because the transmit power is concentrated onto a smaller area on the surface of the sphere, as depicted in Figure 2-2. The power density in the gray ellipse depicting the antenna beam is increased from that of an isotropic antenna. The ratio between the power density for a lossless directional antenna and a hypothetical

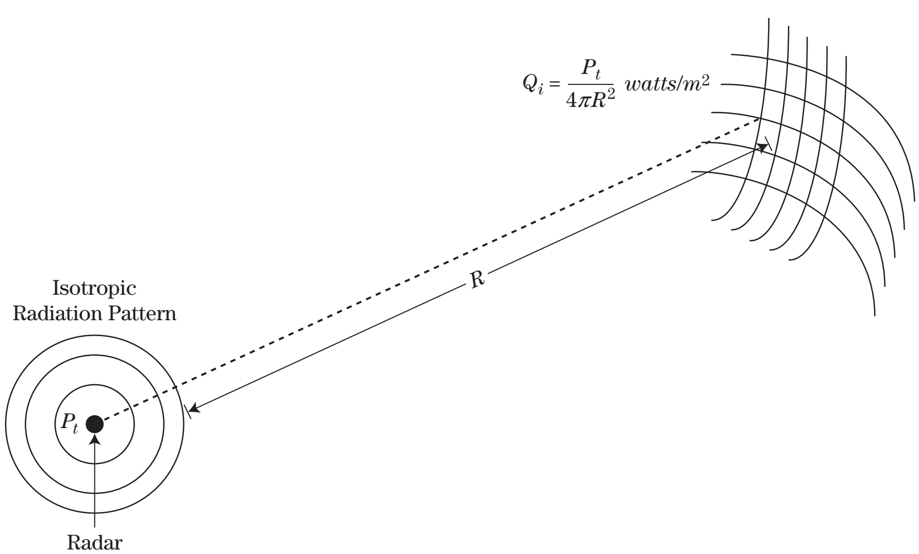

FIGURE 2-1 ■ Power density at range R from the radar transmitter.

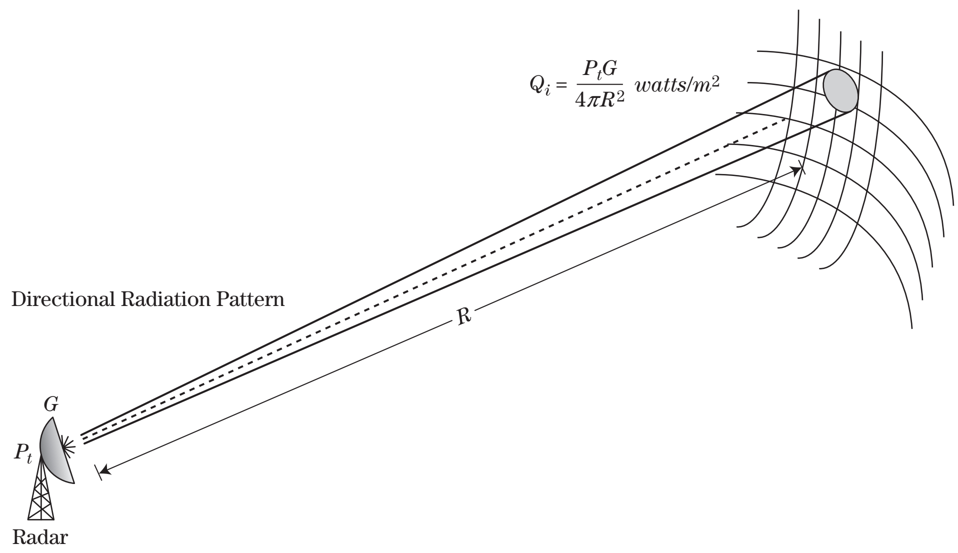

FIGURE 2-2 Power density at range R given transmit antenna gain G_t.

isotropic antenna is termed the *directivity*. The gain, G, of an antenna is the *directivity* reduced by the losses the signal encounters as it travels from the input port to the point at which it is "launched" into the atmosphere [1]. The subscript t is used to denote a transmit antenna, so the transmit antenna gain is G_t. Given the increased power density due to use of a directional antenna,

$$Q_i = \frac{P_t G_t}{4\pi R^2} \tag{2.2}$$

2.3 RECEIVED POWER FROM A TARGET

Next, consider a radar "target" at range R, illuminated by the signal from a radiating antenna. The incident transmitted signal is reflected in a variety of directions, as depicted in Figure 2-3. As described in Chapter 6, the incident radar signal induces time-varying currents on the target so that the target now becomes a source of radio waves, part of which will propagate back to the radar, appearing to be a *reflection* of the illuminating signal. The power *reflected* by the target back toward the radar, P_{refl}, is expressed as the product of the incident power density times and a factor called the *radar cross section* (RCS) σ of the target. The units for RCS are square meters (m^2). The radar cross section of a target is determined by the physical size of the target, the shape of the target, and the materials from which the target is made, particularly the outer surface.[2] The expression for the power reflected back toward the radar, P_{refl}, from the target is

$$P_{refl} = Q_i \sigma = \frac{P_t G_t \sigma}{4\pi R^2} \tag{2.3}$$

[2] A more formal definition and additional discussion of RCS are given in Chapter 6.

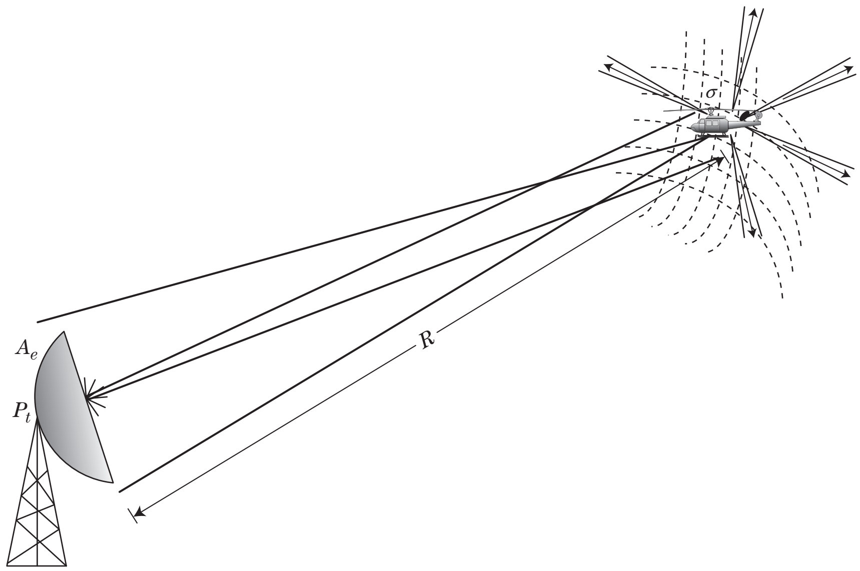

FIGURE 2-3 ■ Power density, Q_r, back at the radar receive antenna.

The signal reflected from the target propagates back toward the radar system over a distance R so that the power density back at the radar receiver antenna Q_r is

$$Q_r = \frac{P_{refl}}{4\pi R^2} \tag{2.4}$$

Combining equations (2.3) and (2.4), the power density of the radio wave received back at the radar receive antenna is given by

$$Q_r = \frac{Q_t \sigma}{4\pi R^2} = \frac{P_t G_t \sigma}{(4\pi)^2 R^4} \tag{2.5}$$

Notice that the radar-target range R appears in the denominator raised to the fourth power. As an example of its significance, if the range from the radar to the target doubles, the received power density of the reflected signal from a target decreases by a factor of 16 (12 dB).

The radar wave reflected from the target, which has propagated through a distance R and results in the power density given by equation (2.5), is received (gathered) by a radar receive antenna having an effective antenna area of A_e. The power received, S, from a target at range R at a receiving antenna of effective area of A_e is found from the power density at the antenna times the effective area of the antenna:

$$S = Q_r A_e = \frac{P_t G_t A_e \sigma}{(4\pi)^2 R^4} \tag{2.6}$$

It is customary to replace the effective antenna area term A_e with the value of receive antenna gain G_r that is produced by that area. Also, as described in Chapter 9, because of the effects of tapering and losses, the *effective* area of an antenna is somewhat less than the physical area, A. As discussed in Chapter 9, as well as in many standard antenna texts, such as [1], the relationship between an antenna gain G and its effective area A_e is given by

$$G = \frac{4\pi \eta_a A}{\lambda^2} = \frac{4\pi A_e}{\lambda^2} \tag{2.7}$$

where η_a is the antenna efficiency. Antenna efficiency is a value between 0 and 1; however, it is seldom below 0.5 and seldom above 0.8.

Solving (2.7) for A_e and substituting into equation (2.6), the following expression for the received power results in

$$S = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4} \tag{2.8}$$

where

- P_t is the peak transmitted power in watts.
- G_t is the gain of the transmit antenna.
- G_r is the gain of the receive antenna.
- λ is the carrier wavelength in meters.
- σ is the mean[3] RCS of the target in square meters.
- R is the range from the radar to the target in meters.

This form is found in many existing standard radar texts, including [2–6].

For many monostatic radar systems, particularly those using mechanically scanned antennas, the transmit and receive antennas gains are the same, so in those cases the two gain terms in (2.8) are replaced by G^2. However, for bistatic systems and in many modern radar systems, particularly those that employ electronically scanned antennas, the two gains are generally different, in which case the preferred form of the radar range equation is that shown in (2.8), allowing for different values for transmit and receive gain.

For a bistatic radar, one for which the receive antenna is not colocated with the transmit antenna, the range between the transmitter and target, R_t, may be different from the range between the target and the receiver, R_r. In this case, the two different range values must be independently specified, leading to the bistatic form of the equation

$$S = \frac{P_t G_t G_r \lambda^2 \sigma_{bistatic}}{(4\pi)^2 R_t^2 R_r^2} \tag{2.9}$$

Though in the following discussions the monostatic form of the radar equation is described, a similar bistatic form can be developed by separating the range terms and using the bistatic radar cross section, $\sigma_{bistatic}$, of the target.

2.4 RECEIVER THERMAL NOISE

In the ideal case, the received target signal, which usually has a very small amplitude, could be amplified by some arbitrarily large amount until it could be visible on a display or within the dynamic range of an analog-to-digital converter (ADC). Unfortunately, as discussed in Chapter 1 and in the introduction to this chapter, there is always an interfering signal described as having a randomly varying amplitude and phase, called *noise*, which is produced by several sources. As discussed in Chapter 1, random noise can be found

[3]The target RCS is normally a fluctuating value, so the mean value is usually used to represent the RCS. The radar equation therefore predicts a mean, or average, value of SNR, since the received power likewise varies.

in the environment, mostly due to solar effects. Noise entering the antenna comes from several sources. Cosmic noise, or galactic noise, originates in outer space. It is a significant contributor to the total noise at frequencies below about 1 GHz but is a minor contributor above 1 GHz. Solar noise is from the sun. Its proximity makes it a significant contributor; however, its effect is reduced by the antenna sidelobe gain, unless the antenna main beam is pointed directly toward the sun. Even the ground is a source of noise, but not as high a level as the sun, and usually enters the receiver through antenna sidelobes.

In addition to antenna noise, thermally agitated random electron motion in the receiver circuits generates a level of random noise with which the target signal must compete. Though there are several sources of noise, the development of the radar range equation in this chapter will assume that the internal noise in the receiver dominates the noise level. This section presents the expected noise power due to the active circuits in the radar receiver. For target *detection* to occur, the target signal must exceed the noise signal and, depending on the statistical nature of the target, sometimes by a significant margin before the target can be detected with a high probability.

Thermal noise power is essentially uniformly distributed over all radar frequencies; that is, its *power spectral density* is constant, or *uniform*. It is sometimes called "white" noise. Therefore, only noise signals with frequencies within the range of frequencies capable of being detected by the radar's receiver will have any effect on radar performance. The range of frequencies for which the radar is susceptible to noise signals is determined by the receiver bandwidth, B. The thermal noise power adversely affecting radar performance will therefore be proportional to B. The noise figure, F, is a measure of the additional noise introduced by the receiver, as described in the following section. The power, P_n, of the thermal noise in the radar receiver is given by [4]

$$P_n = kT_S B = kT_O(F - 1)B \tag{2.10}$$

where

k is Boltzmann's constant (1.38×10^{-23} watt-sec/°K).
T_0 is the standard temperature (290° K).
T_s is the system noise temperature ($T_s = T_0(F - 1)$).
B is the instantaneous receiver bandwidth in Hz.
F is the noise figure of the receiver subsystem (unitless).

The *noise figure* is an alternate method to describe the receiver noise to system temperature, T_s. It is important to note that noise figure is often given in dB; however, it must be converted to linear units for use in equation (2.10).

As can be seen from (2.10), the noise power is linearly proportional to receiver bandwidth. However, the receiver bandwidth cannot be made arbitrarily small to reduce noise power without adversely affecting the target signal. As will be shown in Chapters 8 and 11, for a simple unmodulated transmit signal, the bandwidth of the target's signal in one received pulse is approximated by the reciprocal of the pulse width, τ (i.e., $B \approx 1/\tau$). If the receiver bandwidth is made smaller than the target signal bandwidth, the target power is reduced, and range resolution suffers. If the receiver bandwidth is made larger than the reciprocal of the pulse length, then the signal to noise ratio will suffer. The true optimum bandwidth depends on the specific shape of the receiver filter characteristics. In practice, the optimum bandwidth is usually on the order of $1.2/\tau$, but the approximation of $1/\tau$ is very often used.

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2.5 | SIGNAL-TO-NOISE RATIO AND THE RADAR RANGE EQUATION

When the target signal power, S, is divided by the noise power, P_n, the result is called the signal-to-noise ratio.

The ratio of the signal power to the noise power is S/P_n. For a discrete target, this is the ratio of equation (2.8) to (2.10):

$$SNR = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 (F-1) B} \tag{2.11}$$

Ultimately, the signal-to-interference ratio is what determines radar performance. The interference can be from noise (receiver or jamming) or from clutter or other electromagnetic interference from, for example, motors, generators, ignitions, or cell services. If the power of the receiver thermal noise is N, from clutter is C, and from jamming noise is J, then the SIR is

$$SIR = \frac{S}{N + C + J} \tag{2.12}$$

Although one of these interference sources usually dominates, reducing the SIR to the signal power divided by the dominant interference power, S/N, S/C, or S/J, a complete calculation must be made in each case to see if this simplification applies.

2.6 | MULTIPLE-PULSE EFFECTS

Seldom is a radar system required to detect a target on the basis of a single transmitted pulse. Usually, several pulses are transmitted with the antenna beam pointed in the direction of the (supposed) target. The received signals from these pulses are processed to improve the ability to detect a target in the presence of noise by performing coherent or noncoherent integration (i.e., averaging; see Chapter 15). Many modern radar systems perform spectral analysis (i.e., moving target indication [MTI] or Doppler processing) to improve target detection performance in the presence of clutter. This section describes the effect of such processing. See Chapter 17 for a more complete description of pulse-Doppler processing. Note that the Doppler processing is equivalent to coherent integration insofar as the improvement in SNR is concerned.

Given that the antenna beam has some angular width, as the radar antenna beam scans in angle it will be pointed at the target for more than the time it takes to transmit and receive one pulse. Often the antenna beam is pointed in a given azimuth-elevation angular position, while several (typically on the order of 16 or 20) pulses are transmitted and received. In this case, the integrated SIR is the important factor in determining SNR. If coherent integration processing is employed, (i.e., both the amplitude and the phase of the received signals are used in the processing), the SNR resulting from coherently integrating N pulses, $SNR_c(N)$, is N times the single-pulse SNR, $SNR(1)$:

$$SNR_c(N) = N \cdot SNR(1) \tag{2.13}$$

The process of coherent integration per se is to add the received signal vectors from a sequence of pulses. For a stationary target using a stationary radar, the vectors for a sequence of pulses would be in line and would add head to tail, as described in [4]. If, however, the radar or the target were moving, the phase would be rotating, and the addition of the vectors would result in no larger signal than any one of the original vectors. No improvement in SNR would be realized. To realize an improved SNR, the signal processor would have to "derotate" the vectors before summing. The fast Fourier transform (FFT) process essentially performs this derotation process before adding the vectors. Each FFT filter output represents the addition of several vectors after derotating the vector a different amount for each filter.

A more appropriate form of the RRE when N pulses are coherently combined is thus

$$SNR_c(N) = \frac{P_t G_t G_r \lambda^2 \sigma N}{(4\pi)^3 R^4 k T_0 (F-1) B} \tag{2.14}$$

This form of the RRE is often used to determine the SNR of a system, knowing the number of pulses coherently processed.

Coherent processing uses the phase information when averaging data from multiple pulses. It is also common to use *noncoherent integration* to improve the SNR. Noncoherent integration discards the phase of the individual echo samples, averaging only the amplitude information. It is easier to perform noncoherent integration. In fact, displaying the signal onto a persistent display whose brightness is proportional to signal amplitude will provide noncoherent integration. Even if the display is not persistent, the operator's "eye memory" will provide some noncoherent integration. The integration gain that results from noncoherent integration of N pulses $SNR_{nc}(N)$ is harder to characterize than in the coherent case but for many cases is at least $\sqrt{N}$ but less than N. It is suggested in [4] that a factor of $N^{0.7}$ would be appropriate in many cases.

$$\sqrt{N} \cdot SNR(1) \leq SNR_{nc}(N) \leq N \cdot SNR(1) \tag{2.15}$$

Chapter 15 provides additional detail on noncoherent integration.

2.7 SUMMARY OF LOSSES

To this point, the radar equation has been presented in an idealized form; that is, no losses have been assumed. Unfortunately, the received signal power is usually lower than predicted if the analyst ignores the effects of signal loss. Atmospheric absorption, component resistive losses, and nonideal signal processing conditions lead to less than ideal SNR performance. This section summarizes the losses most often encountered in radar systems and presents the effect on *SNR*. Included are losses due to clear air, rain, component losses, beam scanning, straddling, and several signal processing techniques. It is important to realize that the loss value, if used in the denominator of the RRE as previously suggested, must be a linear (as opposed to dB) value greater than 1.

Often, the loss values are determined in dB notation. It is convenient to sum the losses in dB notation and finally to convert to the linear value. Equation (2.16) provides the total system loss term,

$$L_s = L_t L_{atm} L_r L_{sp} \tag{2.16}$$

where

L_s is the system loss.
L_t is the transmit loss.
L_{atm} is the atmospheric loss.
L_r is the receiver loss.
L_{sp} is the signal processing loss.

The following sections describe the most common of these losses individually.

As a result of incorporating the losses into (2.14), the RRE becomes

$$SNR = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 (F-1) B L_s} \tag{2.17}$$

2.7.1 Transmit Loss

The radar equation (2.14) is developed assuming that all of the transmit power is radiated out an antenna having a gain G. In fact, there is some loss in the signal level as it travels from the transmitter to the antenna, through waveguide or coaxial cable, and through devices such as a circulator, directional coupler, or transmit/receive (T/R) switch. For most conventional radar systems, the loss is on the order of 3 or 4 dB, depending on the wavelength, length of transmission line, and what devices are included. For each specific radar system, the individual losses must be accounted for. The best source of information regarding the losses due to components is a catalog sheet or specification sheet from the vendor for each of the devices. In addition to the total losses associated with each component, there is some loss associated with connecting these components together. Though the individual contributions are usually small, the total must be accounted for. The actual loss associated with a given assembly may be more or less than that predicted. If maximum values are used in the assumptions for loss, then the total loss will usually be somewhat less than predicted. If average values are used in the prediction, then the actual loss will be quite close to the prediction. It is necessary to measure the losses to determine the actual value.

There is some loss between the input antenna port and the actual radiating antenna; however, this term is usually included in the specified antenna gain value provided by the antenna vendor. The analyst must determine if this term is included in the antenna gain term and, if not, must include it in the loss calculations.

2.7.2 Atmospheric Loss

Chapter 4 provides a thorough discussion of the effects of propagation through the environment on the SNR. The EM wave experiences attenuation in the atmosphere as it travels from the radar to the target, and again as the wave travels from the target back to the radar. Atmospheric loss is caused by interaction between the electromagnetic wave and oxygen molecules and water vapor in the atmosphere. Even clear air exhibits attenuation of the EM wave. The effect of this attenuation generally increases with increased carrier frequency; however, in the vicinity of regions in which the wave resonates with the water or oxygen molecules, there are sharp peaks in the attenuation, with relative nulls between these peaks. In addition, fog, rain, and snow in the atmosphere add to the attenuation caused by clear air. These and other propagation effects (diffraction, refraction, and multipath) are discussed in detail in Chapter 4.

Range-dependent losses are normally expressed in units of dB/km. Also, the absorption values reported in the technical literature are normally expressed as one-way loss. For a monostatic radar system, since the signal has to travel through the same path twice, two-way loss is required. In this case, the values reported need to be doubled on a dB scale (squared on a linear scale). For a bistatic radar, the signal travels through two different paths on transmit and receive, so the one-way values are used.

Significant loss can be encountered as the signal propagates through the atmosphere. For example, if the two-way loss through rain is 0.8 dB/km and the target is 10 km away,

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then the rain-induced reduction in SNR is 8 dB compared with the SNR obtained in clear air. The quantitative effect of such a reduction in SNR is discussed in Chapter 3, but to provide a sense of the enormity of an 8 dB reduction in SNR, usually a reduction of 3 dB will produce noticeable system performance reduction.

2.7.3 Receive Loss

Component losses are also present in the path between the receive antenna terminal and the radar receiver. As with the transmit losses, these are caused by receive transmission line and components. In particular, waveguide and coaxial cable, the circulator, receiver protection switches, and preselection filters contribute to this loss value if employed. As with the transmit path, the specified receiver antenna gain may or may not include the loss between the receive antenna and the receive antenna port. All losses up to the point in the system at which the noise figure is specified must be considered. Again, the vendor data provide maximum and average values, but actual measurements provide the best information on these losses.

2.7.4 Signal Processing Loss

Most modern systems employ some form of multipulse processing that improves the single-pulse SNR by the factor *n,* which is the number of pulses in a *coherent processing interval* (CPI), or dwell time. The effect of this processing gain is included in the average power form of the RRE, developed in Section 2.10. If the single-pulse, peak power form is used, then typically a gain term is included in the RRE that assumes perfect coherent processing gain. In either case, imperfections in signal processing are then accounted for by adding a signal processing loss term. Some examples of the signal processing effects that contribute to system loss are beam scan loss, straddle loss (sometimes called scalloping loss), automatic detection constant false alarm rate (CFAR) loss, and mismatch loss. Each of these is described further in the following paragraphs. The discussion describes the losses associated with a pulsed system that implements a fast Fourier transform to determine the Doppler frequency of a detected target.

Beam shape loss arises because the radar equation is developed using the antenna gains (T/R) as if the target is at the center of the beam pattern for every pulse processed during a CPI. In many system applications, such as a mechanically scanning search radar, the target will at best be at the center of the beam pattern for only one of the pulses processed for a given dwell. If the CPI is defined as the time for which the antenna beam scans in angle from the –3 dB point, through the center, to the other –3 dB point, the average loss in signal compared with the case in which the target is always at the beam peak for a typical beam shape is about 1.6 dB. Of course, the precise value depends on the particular shape of the beam as well as the scan amount during a search dwell, so a more exact calculation may be required.

Figure 2-4 depicts a scanning antenna beam, such that the beam scans in angle from left to right. A target is depicted as an aircraft, and five beam positions are shown. (Often there would be more than five pulses for such a scan, but only five are shown here for clarity.) For the first pulse, the target is depicted at 2.8 dB below the beam peak, the second at 0.6 dB, the third at nearly beam center (–0.0 dB), the fourth at –0.3 dB, and the fifth at –2.2 dB. An electronically scanned antenna beam will not scan continuously across a target position during a CPI but will remain at a given fixed angle. In this case, the

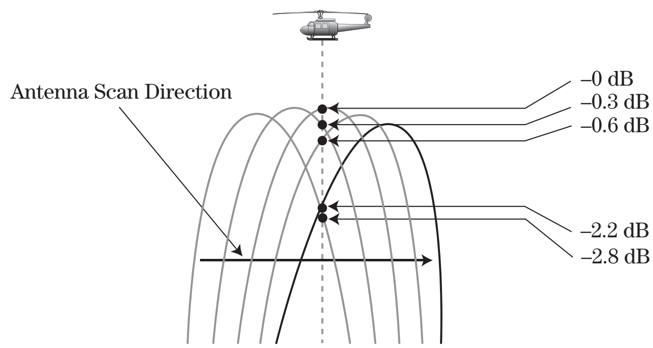

FIGURE 2-4 ■ Target signal loss due to beam scan.

beam shape loss will be constant during the CPI but on average will be the same as for a mechanically scanning antenna during a search frame. Beam shape losses are discussed in more detail in Chapter 9.

In a tracking mode, since the angular position of the target is known, the antenna beam can be pointed directly at the target such that the target is in the center (or at least very close to the center) of the beam for the entire CPI. If this is the case, the SNR for track mode will not be degraded due to the beam shape loss.

The radar system is designed to search for targets in a given volume, defined by the range of elevation and azimuth angles to be considered and the range of distances from the nearest range of interest, R_{min}, to the farthest, R_{max}. Many modern systems also measure the Doppler frequency exhibited by the target. The Doppler frequency can be measured unambiguously from minus half the sample rate to plus half the sample rate. The sample rate for a pulsed system is the pulse repetition frequency (PRF) so the Doppler can be unambiguously determined from $-PRF/2$ to $+PRF/2$.

The system does not determine the range to the target as a continuous value from R_{min} to R_{max}, but rather it subdivides that range extent into contiguous range increments, often called range cells or range bins. The size of any range bin is equivalent to the range resolution of the system. For a simple unmodulated pulse, the range resolution, δR, is $\delta R = c\tau/2$, where τ is the pulse width in seconds, and c is the speed of light. For a 1 microsecond pulse, the range resolution is 150 meters. If the total range is from 1 km to 50 km, there are 327 150 meter range bins to consider.

Likewise, the Doppler frequency regime from $-PRF/2$ to $+PRF/2$ is divided into contiguous Doppler bands by the action of the Doppler filters. The bandwidth of a Doppler filter is on the order of the reciprocal of the well time. A 2 msec dwell will result in 500 Hz filter bandwidth. The total number of Doppler filters is equivalent to the size of the FFT used to produce the results. If analog circuits are used to develop the Doppler measurement, then the number of filters is somewhat arbitrary.

Given that a target may not be at exactly a whole number of range increments from the radar and that the Doppler frequency may not be centered in a Doppler filter, the target may straddle between two range bins or Doppler filters. This leads to some of the target signal being detected in the first of the two range (or Doppler) bins and the remainder of the target signal being detected in the second range (or Doppler) bin. There is an average loss in signal, termed straddle loss.

Straddle loss arises because a target signal is not generally in the center of a range bin or a Doppler filter.. It may be that the centroid of the received target pulse/spectrum is somewhere between two range bins and somewhere between two Doppler filters, reducing

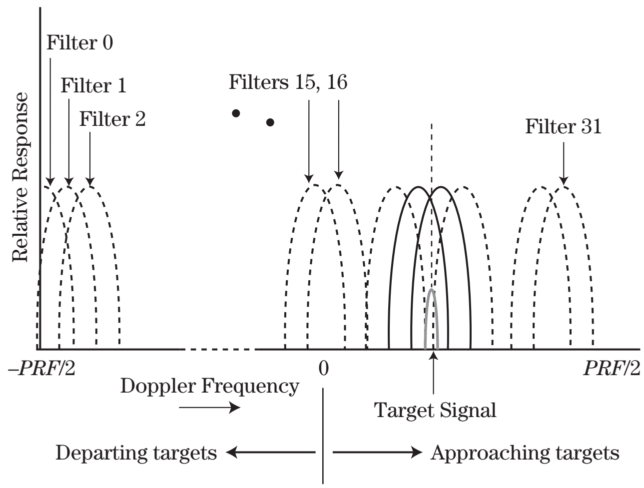

FIGURE 2-5 ■ Doppler filter bank, showing a target straddling two filters.

the target signal power. Figure 2-5 depicts a series of several Doppler filters, ranging from –*PRF*/2 to +*PRF*/2 in frequency. Nonmoving clutter will appear between filters 15 and 16, at 0 Hz (for a stationary radar). A target is depicted at a position in frequency identified by the dashed vertical line such that it is not centered in any filter but instead is "straddling" the two filters shown in the figure as solid lines. A similar condition will occur in the range (time) dimension; that is, a target signal will, in general, appear between two range sample times. The worst-case loss due to range and Doppler straddle depends on a number of sampling and resolution parameters but is usually no more than 3 dB each in range and Doppler. However, usually the average loss rather than the worst case is considered when predicting the SNR. The loss experienced depends on the extent to which successive bins overlap—that is, the depth of the dip between two adjacent bins. Thus, straddle loss can be reduced by oversampling in range and Doppler, which decreases the depth of the "scallop" between bins. Depending on these details, an expected average loss of about 1 dB for range and 1 dB for Doppler is often reasonable. If the system parameters are known, a more rigorous analysis should be performed. Straddle loss is analyzed in more detail in Chapters 14 and 17. As with the beam shape loss, in the tracking mode the range and Doppler sampling can be adjusted so that the target is centered in these bins, eliminating the straddle loss.

Most modern radar systems are designed to automatically detect the presence of a target in the presence of interfering signals, such as atmospheric and receiver noise, intentional interference (jamming), unintentional interference (electromagnetic interference), and clutter. Given the variability of the interfering signals, a CFAR processor might be used to determine the presence of a target. The processor compares the signal amplitude for each resolution cell with a local average, or mean of the surrounding cells, which ostensibly contain only interference signals. A threshold is established at some level (several standard deviations) above such an average to maintain a predicted average rate of false alarm. If the interference level is constant and known, then an optimum threshold level can be determined that will maintain a fixed probability of false alarm, P_{FA}. However, because the interfering signal is varying, the interference may be higher than the mean is some region of the sample space and may be lower than the mean in other regions. To avoid a high P_{FA} in any region, the threshold will have to be somewhat higher than the optimum setting. This means that the probability of detection, P_D, will be somewhat lower than optimum. The consequence is that the SNR must be higher than that required for an optimum detector for a given P_D. The SNR is not increased due to this effect, but it is considered to be a *loss*. Such a loss in detection performance is called a *CFAR loss* and is on the order of 1 to 2 dB for most standard conditions. Chapter 16 provides a complete discussion of the operation of a CFAR processor and its attendant losses.

The SNR is estimated given a matched filter in the receiver. A matched filter is a receiver frequency response designed to maximize the output SNR; see Chapters 14 and 20 for a detailed discussion of matched filters. Thus, it is assumed that most of the target signal comes through the receiver filter and that the noise bandwidth is no more than that required for a given target signal bandwidth. For a simple (unmodulated) pulse, this occurs when the noise bandwidth is about $1.2/\tau$, depending on the spectral shape of the signal and the particular implementation of the receiver filters. If the filter bandwidth is any wider than this, though some additional target signal increase is experienced, the noise power increases proportionally with the increase of the bandwidth. That is, if the bandwidth doubles, the noise power doubles but the signal power increases only marginally, reducing the SNR. This decrease in SNR is the mismatch loss, resulting from a receiver bandpass characteristic that is not optimally selected for the transmitted pulse shape.

For a pulse compression system, the matched filter condition is obtained only when there is no Doppler frequency offset on the target signal or when the Doppler shift is compensated in the processing. If neither of these is the case, a Doppler mismatch loss is usually experienced.

2.8 SOLVING FOR OTHER VARIABLES

2.8.1 Range as a Dependent Variable

An important analysis is to determine the detection range, R_{det}, at which a given target RCS can be detected with a given SNR. In this case, solving equation (2.17) for R yields

$$R_{det} = \left[\frac{P_t G_t G_r \lambda^2 \sigma N}{(4\pi)^3 \, SNR k T_0 (F-1) B L_s} \right]^{\frac{1}{4}} \tag{2.18}$$

In using (2.18), though, bear in mind that some of the losses in L_s (primarily atmospheric attenuation) are range-dependent.

2.8.2 Solving for Minimum Detectable RCS

Another important analysis is to determine the minimum detectable radar cross section, σ_{min}. This calculation is based on assuming that there is a minimum SNR, SNR_{min}, required for reliable detection (see Chapter 15). Substituting $SNR_{min}$for SNR and solving (2.17) for radar cross section yields

$$\sigma_{min} = SNR_{min} \frac{(4\pi)^3 \, R^4 k T_0 (F-1) B L_s}{P_t G_t G_r \lambda^2 N} \tag{2.19}$$

Clearly, equation (2.16) could be solved for any of the variables of interest. However, these provided forms are most commonly used.

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2.9 DECIBEL FORM OF THE RADAR RANGE EQUATION

Many radar systems engineers use the previously presented form of the radar equation, which is given in linear space. That is, the equation consists of a set of values that describe the radar parameters in, for example, watts, seconds, or meters, and the values in the numerator are multiplied and divided by the product of the values in the denominator. Other radar systems engineers prefer to convert each term to the dB value and to add the numerator terms and subtract the denominator terms, resulting in SNR being expressed directly in dB. The use of this form of the radar equation is based strictly on the preference of the analyst. Many of the terms in the SNR equation are naturally determined in dB notation, and many are determined in linear space, so in either case some of the terms must be converted from one space to the other. The terms that normally appear in dB notation are antenna gains, RCS, noise figure, and system losses. It remains to convert the remaining values to dB equivalents and then to proceed with the summations. Equation (2.20) demonstrates the dB form of the RRE shown in equation (2.17).

$$\begin{aligned} SNR_c\,[\text{dB}] = {} & 10\log_{10}(P_t) + G_t\,[\text{dB}] + G_r\,[\text{dB}] + 20\log_{10}(\lambda) + \sigma[dBsm] \\ & +10\log_{10}(N) - 33 - 40\log_{10}(R) - (-204)\,[\text{dBW / Hz}] \\ & -(F-1)\,[\text{dB}] - 10\log_{10}(B)\,\,[\text{dBHz}] - L_s\,[\text{dB}] \end{aligned} \tag{2.20}$$

In the presentation in (2.20) the constant values (e.g., π, kT_0) have been converted to the dB equivalent. For instance, $(4\pi)^3 \approx 1{,}984$, and $10\log_{10}(1{,}984) \cong 33$ dB. (since this term is in the denominator, it results in –33 dB in equation [2.19]). The (–204) [dBW/Hz] term results from the product of k and T_0. To use orders of magnitude that are more appropriate for signal power and bandwidth in the radar receiver, this is equivalent to –114 dBm/MHz. Remembering this value makes it easy to modify the result for other noise temperatures, the noise figure, and the bandwidth in MHz. In addition to the simplicity associated with adding and subtracting, the dB form lends itself more readily to tabulation and spreadsheet analysis.

2.10 AVERAGE POWER FORM OF THE RADAR RANGE EQUATION

Given that the radar usually transmits several pulses and processes the results of those pulses to detect a target, an often used form of the radar range equation replaces the peak power, number of pulses processed, and instantaneous bandwidth terms with average power and dwell time. This form of the equation is applicable to all coherent multipulse signal processing gain effects.

The average power, P_{avg}, form of the RRE can be obtained from the peak power, P_t, form with the following series of substitutions:

$$T_d = \text{dwell time} = N \cdot PRI = N/PRF \tag{2.21}$$

where PRI is the interpulse period (time between transmit pulses), and PRF is the pulse repetition frequency.

Solving (2.21) for N

$$N = T_d \cdot PRF \tag{2.22}$$

$$\text{Duty cycle} = \tau \cdot PRF \tag{2.23}$$

$$P_{avg} = P_t \cdot (\text{duty cycle}) = P_t \cdot (\tau \cdot PRF) \tag{2.24}$$

For a simple (unmodulated) pulse of width τ, the optimum receiver bandwidth, B, is

$$B = 1/\tau \tag{2.25}$$

Combining (2.22), (2.24), and (2.25) and solving for P_t gives

$$P_t = P_{avg} T_d B/N \tag{2.26}$$

Substituting P_t in (2.26) for P_t in (2.17) gives

$$SNR_c = \left(\frac{P_{avg} T_d B}{N}\right) \frac{G_t G_r \lambda^2 \sigma N}{(4\pi)^3 R^4 k T_0 (F-1) L_s B} = \frac{P_{avg} T_d G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 (F-1) L_s} \tag{2.27}$$

In this form of the equation, the average power–dwell time terms provide the energy in the processed waveform, while the $kT_0(F-1)$ terms provide the noise energy. Assuming that all of the conditions related to the substitutions described in (2.21) through (2.25) are met—that is, the system uses coherent integration or equivalent processing during the dwell time and the receiver bandwidth is matched to the transmit bandwidth—the average power form of the radar range equation provides some valuable insight for SNR. In particular, the SNR for a system can be adjusted by changing the dwell time without requiring hardware changes, except that the signal/data processor must be able to adapt to the longest dwell. Often, for a coherent radar in which N pulses are coherently processed, the dwell time, T_d, is called the coherent processing interval.

2.11 PULSE COMPRESSION: INTRAPULSE MODULATION

The factor of N in equation (2.17) is a form of signal processing gain resulting from coherent integration of multiple pulses. Signal processing gain can also arise from processing pulses with intrapulse modulation. Radar systems are sometimes required to produce a given probability of detection, which would require a given SNR, while at the same time maintaining a specified range resolution. When using simple (unmodulated) pulses, the receiver bandwidth is inversely proportional to the pulse length τ, as discussed earlier. Thus, increasing the pulse length will increase the SNR. However, range resolution is also proportional to τ, so the pulse must be kept short to meet range resolution requirements. A way to overcome this conflict is to maintain the average power by transmitting a wide pulse while maintaining the range resolution by incorporating a wide bandwidth in that pulse—wider than the reciprocal of the pulse width. This extended bandwidth can be achieved by incorporating modulation (phase or frequency) within the pulse. Proper matched filtering of the received pulse is needed to achieve both goals. The use of intrapulse modulated waveforms to achieve fine-range resolution while maintaining high average power is called *pulse compression*.

As presented in Chapter 20, the appropriate form of the radar range equation for a system using pulse compression is

$$SNR_{pc} = SNR_u \tau \beta \tag{2.28}$$

where

SNR_{pc} is the signal-to-noise ratio for a modulated (pulse compression) pulse.
SNR_u is the signal-to-noise ratio for an unmodulated pulse.
τ is the pulse length.
β is the pulse modulation bandwidth.

Substituting this into (2.17) gives

$$SNR_{pc} = \frac{P_t G_t G_r \lambda^2 \sigma\ N}{(4\pi)^3\ R^4 k T_0 (F-1) B L_s} \tau \beta \tag{2.29}$$

Using (2.29) and the substitution developed in equation (2.26) for the average power form of the radar range equation, the same substitutions can be made, resulting in

$$SNR_{pc} = \left(\frac{P_{avg} T_d}{N\tau}\right) \frac{G_t G_r \lambda^2 \sigma\ N}{(4\pi)^3\ R^4 k T_0 (F-1) B L_s} \tau \beta = \frac{P_{avg} T_d G_t G_r \lambda^2 \sigma}{(4\pi)^3\ R^4 k T_0 (F-1) L_s} \tag{2.30}$$

This equation demonstrates that the average power form of the radar range equation is appropriate for a modulated pulse system as well as for a simple pulse system. As with the unmodulated pulse, appropriate use of the average power form requires that coherent integration or equivalent processing is used during the dwell time and that matched filtering is used in the receiver.

2.12 A GRAPHICAL EXAMPLE

Consider an example of a hypothetical radar system SNR analysis in tabular form and in graphical form. Equation (2.27) is used to make the SNR_c calculations. The example plot of SNR_c as a function of target range shown in Figure 2-6 is a ground- or air-based radar system with the following characteristics:

Transmitter:	10 kilowatt peak power
Frequency:	9.4 GHz
Pulse width:	0.1 microseconds
PRF:	1 kilohertz
Antenna:	0.8 meter diameter circular antenna (An efficiency, η, of 0.6 is to be used to determine antenna gain.)
Target RCS:	0 dBsm, –10 dBsm
Processing dwell time	7.62 milliseconds
Receiver noise figure:	2.5 dB
Transmit losses:	3.1 dB
Receive losses:	2.4 dB
Signal processing losses:	3.2 dB
Atmospheric losses:	0.16 dB/km (one way)
Target range:	1 to 100 km

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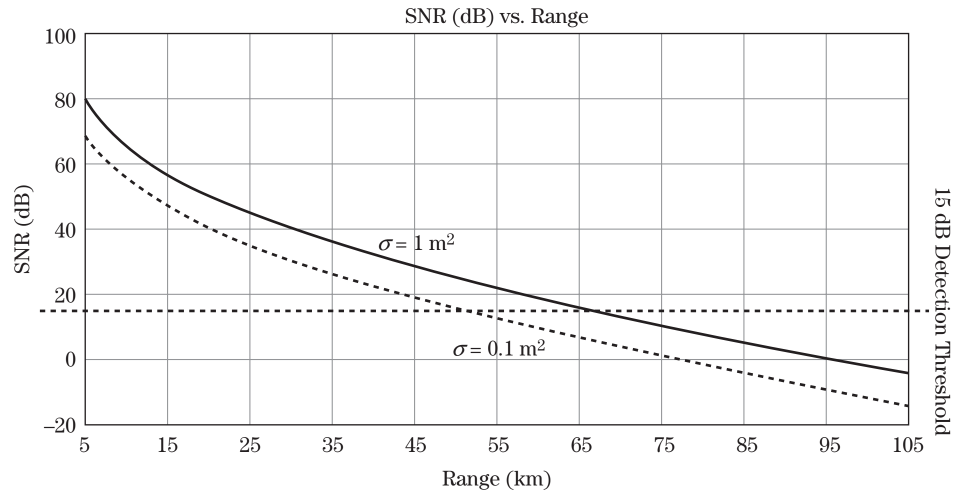


FIGURE 2-6 ■ Graphical solution to radar range equation.

It is customary to plot the SNR in dB as a function of range from the minimum range of interest to the maximum range of interest. Figure 2-6 is an example of a plot for two target RCS values resulting from the given parameters. If it is assumed that the target is reliably detected at an SNR of about 15 dB, then the 1 m^2 target will be detectable at a range of approximately 65 km, whereas the 0.1 m^2 target will be detectable at approximately 49 km.

Once the formulas for this plotting example are developed in a spreadsheet program such as Microsoft Excel, then it is relatively easy to extend the analysis to plotting probability of detection (see Chapter 3) and tracking measurement precision (Chapter 17) as functions of range, since these are dependent primarily on SNR and additional fixed parameters.

2.13 CLUTTER AS THE TARGET

Though the intent is usually to detect a discrete target in the presence of noise and other interference, there are often unintentional signals received from other objects in the antenna beam. Unintentional signals can result from illuminating clutter, which can be on the surface of the earth, either on land or sea, or in the atmosphere, such as rain and snow. For surface clutter, the area illuminated by the radar antenna beam pattern, including the sidelobes, determines the signal power. For atmospheric clutter, the volume is defined by the antenna beamwidths and the pulse length. The importance of the radar equation is to determine the target SIR, given that the interference is surface or atmospheric clutter. In the case of either, the ratio is determined by dividing the target signal, S, by the clutter signal, S_c, to produce the target-to-clutter ratio, SCR. In many cases, all of the terms in the radar equation cancel except for the RCS (σ or σ_c) terms, resulting in

$$SCR = \frac{\sigma}{\sigma_c} \tag{2.31}$$

In some cases, as with a ground mapping radar or weather radar, the intent is to detect these objects. In other cases, the intent is to detect discrete targets in the presence of these interfering signals. In either case, it is important to understand the signal received from these "clutter" regions. The use of the RRE for the signal resulting from clutter is summarized by substituting the RCS of the clutter cell into the RRE in place of the target

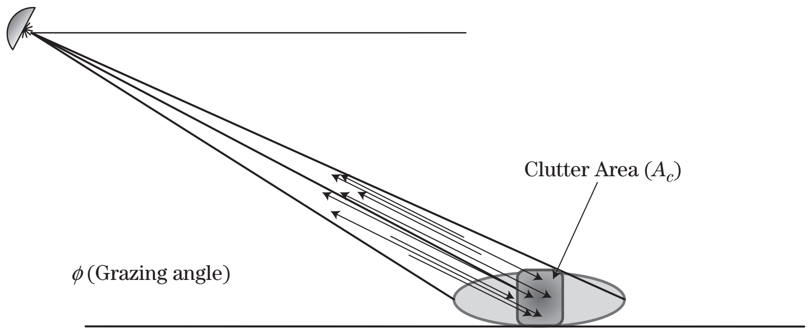


FIGURE 2-7 ■ Area (surface) clutter.

RCS. Chapter 5 describes characteristics and statistical behavior of clutter in detail. A summary is provided here.

2.13.1 Surface Clutter

The radar cross section value for a surface clutter cell is determined from the average reflectivity, σ^0, of the particular clutter type, in square meters per unit area, times the area of the clutter cell, A_c, illuminated by the radar:

$$\sigma_{cs} = A_c \sigma^0 \tag{2.32}$$

where

σ_{cs} is the surface clutter radar cross section.
A_c is the area of the illuminated (ground or sea surface) clutter cell.
σ^0 is the surface backscatter coefficient (average reflectivity per unit area).

Chapter 5 includes a thorough discussion of an analysis of the area calculation, which depends on the radar scenario. Figure 2-7 depicts the area of a clutter cell illuminated on the surface. The clutter consists of a multitude of individual reflecting objects (e.g., rocks, grass, dirt mounds, twigs, branches), often called *scatterers*. The resultant of these many *scatterers* is a single net received signal back at the radar receiver.

2.13.2 Volume Clutter

The radar cross section value for volumetric clutter cell is determined from the average reflectivity of the particular clutter type per unit volume, η, times the volume of the clutter cell, V_c, illuminated by the radar:

$$\sigma_{cv} = V_c \eta \tag{2.33}$$

where

σ_{cv} is the volume clutter radar cross section.
V_c is the volume of the illuminated clutter cell.
η is the volumetric backscatter coefficient (average reflectivity per unit volume).

The volume, V_c, of the cell illuminated by the radar is depicted in Figure 2-8. Chapter 5 includes a thorough discussion of the volume calculation, which depends on the radar scenario. The clutter in Figure 2-8 consists of a multitude of individual reflecting objects (e.g., rain, fog droplets). The resultant of these many *scatterers* is a single net received signal back at the radar receiver.

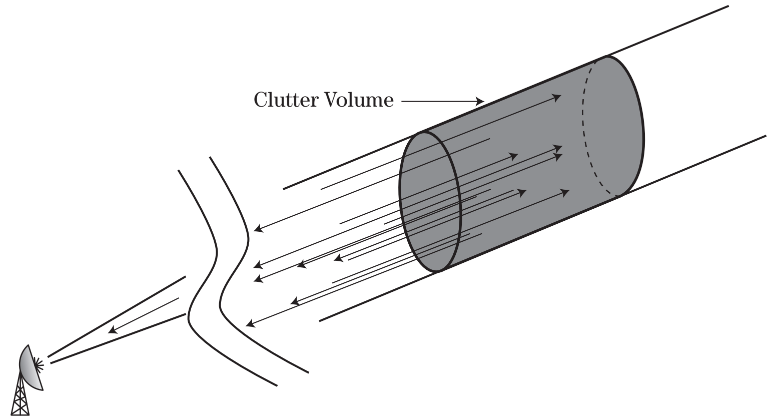


FIGURE 2-8 ■ Volumetric (atmospheric) clutter.

2.14 ONE-WAY (LINK) EQUATION

To this point, the components of the interfering signals discussed have been receiver thermal noise and clutter. In many defense-oriented radar applications, it is expected that the radar system will encounter intentional jamming. Jamming signals are one of two varieties: noise and false targets. The effect of noise jamming is to degrade the SIR of target signals in the radar receiver to delay initial detection or to degrade tracking performance. In many cases, intentional noise jamming is the most limiting interference. The intent of false target jamming is to present so many target-like signals that the radar processor is overloaded or to create false track files. In any case, the power received from a jammer is required to determine its effect on radar performance. Since the signal from the jammer to the radar has to propagate in only one direction, a simplification of the radar equation for one-way propagation is valuable.

The first step is to determine the effective radiated power out of the jammer antenna, and the power density at a distance, R_{jr}, from the jammer radiating antenna to the radar. The jammer signal power density, Q_j, in watts per square meter at a distance (range) R from a transmitting source can be determined using equation (2.1). Q_j depends on the jammer's transmitted power, P_j; the transmit path losses in the jammer equipment, L_{tj}; the range from the jammer to the radar, R_{jr}; the gain of the jammer transmitting antenna, G_j; and the losses through the propagation medium, L_{atm}. Usually, the antenna gain includes the losses between the antenna input terminal and the antenna. The power density, Q_j, from an isotropic radiating jammer source is the total power, reduced by losses and distributed uniformly over the surface area of a sphere of radius R_{jr}

$$Q_j = \frac{P_j}{4\pi R_{jr}^2 L_{tj} L_{atm}} \tag{2.34}$$

The power density given in (2.33) is increased by the effects of a jammer antenna, which concentrates the energy in a given direction. The power density, Q_j, at the center of the beam for a radiating source with an attached jammer antenna of gain, G_j, is

$$Q_j = \frac{P_j G_j}{4\pi R_{jr}^2 L_{tj} L_{atm}} \tag{2.35}$$

The jammer antenna peak gain, G_j, accounts for the fact the transmitted radio waves are "focused" by the antenna in a given direction, thus increasing the power density in that direction.

Next, consider the radar receiving system at a distance, R_{jr}, from the jammer to the radar. Such a receiving system will have a directive antenna with an effective area of A_e and will have receiver component losses, L_r. The total power received at the radar from the jammer, P_{rj}, is

$$P_{rj} = Q_j A_e = \frac{P_j G_j A_e}{4\pi R_{jr}^2 L_{tj} L_{atm} L_r} \tag{2.36}$$

Equation (2.36) is the *one-way link equation*. It is very useful in predicting the performance of a one-way system such as a communications system or, for a radar, a jammer.

Often, the antenna gain is known instead of the effective area. In this case, using equation (2.7) the area can be replaced by

$$A_e = \frac{G_{rj}\lambda^2}{4\pi} \tag{2.37}$$

which results in

$$P_{rj} = \frac{P_j G_j G_{rj} \lambda^2}{(4\pi)^2 R_{jr}^2 L_{tj} L_{atm} L_r} \tag{2.38}$$

where G_{rj} is the gain of the radar antenna in the direction of the jammer. This is important, since the radar antenna is not necessarily pointed directly at the jammer. In this case, the main beam gain is not appropriate, but the sidelobe gain is. The sidelobes are not easily determined until the antenna is tested in a high-quality environment.

2.15 | SEARCH FORM OF THE RADAR RANGE EQUATION

Section 1.8.1 and Figure 1-23 depict a scanning antenna being used to scan a volume. Analysis of such a system designed to search a given solid angular volume, Ω, in a given search frame time, T_{fs}, is often made easier by using the so-called search form of the radar equation. The predominant figure of merit for such as system is the *power-aperture product* of the system. This section derives and describes this form of the radar equation.

The total time required to search a volume, T_{fs}, is easily determined from the number of beam positions required, M, multiplied by the dwell time required at each of these positions, T_d:

$$T_{fs} = M \cdot T_d \tag{2.39}$$

The number of beam positions required is the solid angular volume to be searched, Ω, divided by the solid angular volume of the antenna beam, which is approximately the product of the azimuth and elevation beamwidths[4]:

$$M = \Omega / \theta_{az}\theta_{el} \tag{2.40}$$

[4]There are $(180/\pi)^2 \cong 3{,}282.8$ square degrees in a steradian.

If the estimated beamwidth is about $1.22\lambda/D$, then the solid angle $\theta_{az}\theta_{el}$ is about $1.5\lambda^2/D^2$. The area, A, of a circular antenna is $\pi D^2/4$, and, from equation (2.8) the effective antenna area, depending on the weighting function and shape, is about $0.6\pi D^2/4$, leading to

$$\theta_{az}\theta_{el} \approx \lambda^2/A_e \tag{2.41}$$

The antenna gain, G, is related to the effective area by

$$G = 4\pi A_e/\lambda^2 \tag{2.42}$$

Using the previous substitutions into (2.17), it can be shown that the resulting SNR can be expressed as

$$SNR = (P_{avg}A_\varepsilon)\frac{1}{4\pi k T_0(F-1)L_s}\left(\frac{\sigma}{R^4}\right)\left(\frac{T_{fs}}{\Omega}\right) \tag{2.43}$$

By substituting the minimum SNR required for reliable detection, SNR_{min}, for SNR and arranging the terms differently, the equation can be repartitioned to place the "user" terms on the right side and the system designer terms on the left side.

$$\frac{P_{avg}A_e}{L_s T_0(F-1)} \geq SNR_{min}4\pi k\left(\frac{R^4}{\sigma}\right)\left(\frac{\Omega}{T_{fs}}\right) \tag{2.44}$$

Since these modifications to the RRE are derived from equation (2.17), the assumptions of coherent integration and matched filtering in the receiver apply.

This *power-aperture form of the RRE* provides a convenient way to partition the salient radar parameters (P_{avg}, A_e, L_s, and F) given the requirement to search for a target of RCS σ at range R over a solid angle volume Ω in time T_{fs}. Since it is derived from the average power form of the basic RRE (2.17), it is applicable for any waveform, whether pulse compression is used, and for any arbitrary length dwell time. It does assume that the entire radar resources are used for search; that is, if the system is a multifunction radar then a loss must be included for the time the radar is in the track mode or is performing some function other than search.

2.16 | TRACK FORM OF THE RADAR RANGE EQUATION

With modern radar technology rapidly evolving toward *electronically scanned arrays* (ESAs, or phased arrays), with additional degrees of freedom, target tracking systems can track multiple targets simultaneously. As with the search form of the radar equation, analysis of a system designed to track multiple targets with a given precision is described in terms of the power-aperture cubed, or, equivalently, the power-aperture-gain form of the radar range equation. This variation is also called the track form of the RRE. These forms are used in cases in which the number of targets being tracked and the track precision or the required SNR are known.

Recalling (2.7), the relationship between an antenna's gain, G, and its effective area, A_e, is

$$G \cong \frac{4\pi A_e}{\lambda^2} \tag{2.45}$$

The approximate beamwidth, *BW*, of an antenna in degrees is [7]

$$\theta_3, \phi_3 \text{ (degrees)} \cong 70\frac{\lambda}{D} \tag{2.46}$$

Since there are $180/\pi$ degrees in a radian, this is equivalent to

$$\theta_3, \phi_3 \text{ (radians)} \cong 1.22\frac{\lambda}{D} \tag{2.47}$$

Of course, there is some variation in this estimate due to specific design parameters and their effects, but this approximation serves as a good estimate. Also, the effective area for a circular antenna of diameter D is [1]

$$A_e \cong \frac{0.6\pi D^2}{4} \tag{2.48}$$

For a more general elliptical antenna, it is

$$A_e \cong \frac{0.6\pi D_{major} D_{minor}}{4} \tag{2.49}$$

where D_{major} and D_{minor} are the major and minor axes of the ellipse, respectively.

From these equations, the solid angle beamwidth is

$$\theta^2 \cong \frac{\lambda^2\pi}{4A_e} \tag{2.50}$$

As described in Chapter 18, a common expression for the estimated tracking noise with a precision σ_θ (standard deviation of the tracking measurement noise) is

$$\sigma_\theta \cong \frac{\theta}{k_m\sqrt{2SNR}} \tag{2.51}$$

where k_m is a tracking system parameter. Substituting (2.50) into (2.51) and solving for SNR gives

$$SNR \cong \frac{\pi\lambda^2}{8A_e k_m^2 \sigma_\theta^2} \tag{2.52}$$

Given a requirement to track N_t targets, each at an update rate of r measurements per second, the dwell time T_d per target is

$$T_d = \frac{1}{r \cdot N_t} \tag{2.53}$$

Finally, substituting equations (2.45), (2.52), and (2.53) into (2.17) and rearranging terms gives

$$\frac{\pi\lambda^2}{8k_m^2\sigma_\theta^2} = \frac{P_{avg}A_e^3\sigma}{4\pi r N_t \lambda^2 k T_0 (F-1) L_s R^4} \tag{2.54}$$

As with the search form of the RRE, the terms are arranged so that the "user" terms are on the right and the "designer" terms are on the left, providing

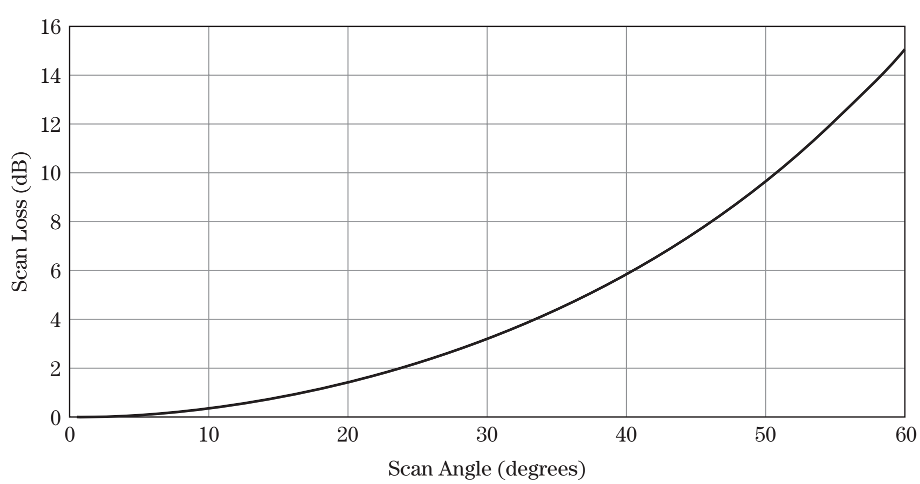
$$\frac{P_{avg}A_e^3k_m^2}{\lambda^4 L_s T_0 (F-1)} = \left(\frac{\pi^2}{2}\right)\left(\frac{k r N_t R^4}{\sigma \cdot \sigma_\theta^2}\right)\left(\frac{1}{\cos^5(\theta_{scan})}\right) \tag{2.55}$$

This form of the RRE shows that, given N_t targets of RCS σ at range R to track, each at rate r and with a precision σ_θ, the power-aperture cubed of the radar becomes the salient determinant of performance. Coherent integration and matched filtering are assumed, since these developments are based on equation (2.17).

Equation (2.55) also introduces a cosine^5 term that has not been seen thus far. This term accounts for the beam-broadening effect and the gain reduction that accompanies the scanning of an electronically scanned antenna to an angle of θ_{scan} from array normal. To a first order, the beamwidth increases as the beam is scanned away from array normal by the cosine of the scan angle due to the reduced effective antenna along the beam-pointing direction. The gain is also reduced by the cosine of the scan angle due to the reduced effective antenna and by another cosine factor due to the off-axis gain reduction in the individual element pattern, resulting in a net antenna gain reduction by a factor of $\cos^2(\theta_{scan})$. SNR is reduced by the product of the transmit and receive antenna gains, thus squaring the reduction to a factor of $\cos^4(\theta_{scan})$. Therefore, to maintain a constant angle precision, the radar sensitivity needs to increase by $\cos^5(\theta_{scan})$: $\cos^4$ due to gain effects, and another cosine factor due to beam broadening. This term is an approximation, because the individual element pattern is not strictly a cosine function. However, it is a good approximation, particularly at angles beyond about 30 degrees. Additional details on the effect of scanning on the gain and beamwidth of ESAs are given in Chapter 9.

Figure 2-9 is a plot of the loss associated with scanning an electronically scanned beam. Because of the wider antenna beam and lower gain, the radar energy on target must increase by this factor. Compared with a target located broadside to the array, a target at a 45 degree scan angle requires increased energy on the order of 7 dB to maintain the same tracking precision; at 60 degrees, the required increase is 15 dB. These huge equivalent losses can be overcome by the geometry of the problem or by changing the radar waveform at the larger scan angles. For example, once a target is in track, it will be close to the antenna normal (for an airborne interceptor); for a fixed or ship-based system, the scan loss is partially offset by using longer dwell times at the wider scan angles. Also, for a target in track, straddle losses (see Chapters 14, 17, and 18) are reduced because the target is likely to be near the center of the beam and also to be nearly centered in the range and Doppler bins. Nonetheless, the system parameters must be robust enough and the processor must be adaptable enough to support shorter than average dwell times at near-normal scan angles to allow time for longer dwells at the extreme angles to offset large scan losses.

FIGURE 2-9 ■ Scan loss versus angle for an electronically scanned antenna beam.



Occasionally, another form of the radar equation as it relates to a tracking system is encountered. Beginning with equation (2.11), repeated here for convenience,

$$SNR = \frac{P_t G_t G_r \lambda^2 \sigma}{(4\pi)^3 R^4 k T_0 (F-1) B L_s} \tag{2.56}$$

making the substitution P_{avg} = average power = $P_t \cdot \tau \cdot PRF$ and solving for P_{avg} gives

$$P_{avg} = \frac{SNR\,(4\pi)^3 R^4 k T_0 (F-1) B L_s \tau PRF}{G_t G_r \sigma \lambda^2} \tag{2.57}$$

Substituting from equation (2.45) of the antenna gain gives

$$P_{avg} = \frac{SNR \cdot 4\pi R^4 k T_0 (F-1) L_s PRF \lambda^2}{\sigma A_e^2} \tag{2.58}$$

or, rearranged,

$$\frac{P_{avg} A_e^2}{L_s (F-1) \lambda^2} = \frac{SNR \cdot 4\pi R^4 k T_0 PRF}{\sigma} \tag{2.59}$$

Equation (2.59) is the power-aperture squared form of the radar range equation. This form is most useful when the required SNR is known, whereas equation (2.55) is used when the required tracking precision, σ_θ, is known. The two forms (2.55) and (2.59) are equivalent when the substitutions associated with the relationship between antenna dimensions, SNR, and tracking precision are incorporated.

A final form, also sometimes encountered is found by replacing one of the A_e terms on the left side of (2.59) with its equivalent in terms of gain,

$$A_e = \frac{G\lambda^2}{4\pi} \tag{2.60}$$

resulting in

$$\frac{P_{avg} A_e G}{L_s (F-1)} = \frac{SNR \cdot (4\pi)^2 R^4 k T_0 PRF}{\sigma} \tag{2.61}$$

which does not include wavelength, λ. Clearly some of the terms are dependent on λ, such as L and F. Equation (2.61) is often called the power-aperture-gain form of the RRE.

2.17 | SOME IMPLICATIONS OF THE RADAR RANGE EQUATION

Now that the reader is somewhat familiar with the basic RRE, its use in evaluating radar detection range performance can be explored.

2.17.1 Average Power and Dwell Time

Considering equation (2.27), the average power form of the RRE, it can be seen that for a given hardware configuration that "freezes" the P_{avg}, G, λ, F, and L_s, the dwell time, T_d, can easily be changed without affecting the hardware design. This directly impacts the SNR. For example, doubling the dwell time increases the SNR by 3 dB. As Chapter 3 shows, this increase in SNR improves the detection statistics; that is, the probability of detection, P_D, for a given probability of false alarm, P_{FA}, will improve.

For an electronically scanned antenna beam, the radar received signal power and therefore the SNR degrade as the beam is scanned away from normal to the antenna surface, as was seen in the context of equation (2.55) and Figure 2-9. This reduction in SNR can be recovered by adapting the dwell time to the antenna beam position. For example, whereas a mechanically scanned antenna beam might have a constant 2 msec dwell time, for the radar system using an ESA of similar area, the dwell can be adaptable. It might be 2 msec near normal (say, 0 to 30 degrees), 4 msec from 30 to 40 degrees scan angle, and 8 msec from 40 to 45 degrees. Depending on the specific design characteristics of the ESA, it might have lower losses than the mechanically scanned antenna system so that the dwell times for the various angular beam positions could be less.

2.17.2 Target RCS Effects

Much is being done today to reduce the radar cross section of radar targets, such as missiles, aircraft, land vehicles (tanks and trucks), and ships. The use of radar-absorbing material and target shape modifications can produce a significantly lower RCS compared with conventional designs. This technology is intended to make the target "invisible" to radar. In fact, the change in radar detection range performance is subtle for modest changes in target RCS. For example, if the RCS is reduced by 3 dB, the detection range decreases by only about 16% to 84% of the baseline value. To reduce the effective radar detection range performance by half, the RCS must be reduced by a factor of 16, or 12 dB. Thus, an aggressive RCS reduction effort is required to create significant reductions in radar detection range. Basic concepts of RCS reduction are introduced in Chapter 6.

2.18 FURTHER READING

Most standard radar textbooks have a section that develops the radar range equation with a similar yet different approach. A somewhat more detailed approach to development of the peak power form of the RRE can be found in Chapter 4 of Barton et al. [5], Difranco and Rubin [8], and Sullivan [9]. A further discussion of the energy (average power) form is found in the same references. It is sometimes appropriate to present the results of RRE analysis in a tabular form. One form that has been in use since about 1969 is the Blake chart [4]. A more complete discussion of the various sources of system noise is presented in Chapter 4 of Blake [3]. A comprehensive discussion of the various RRE loss terms is also presented in many of the aforementioned texts as well as in Nathanson [6] and Barton et al. [5].

2.19 REFERENCES

[1] Johnson, R.C., *Antenna Engineering Handbook*, 3d ed., McGraw Hill, New York, 1993, Ch. 46.

[2] Krause, J.D., *Antennas,* McGraw-Hill, New York, 1988, p. 27.

[3] Eaves, J.L., and Reedy, E.K., *Principles of Modern Radar,* Van Nostrand Reinhold, New York, 1987, equation (1–16).

[4] Blake, L.V., *Radar Range-Performance Analysis*, Artech House, Dedham, MA, 1986.

[5] Barton, D.K., Cook, C.E., and Hamilton, P., *Radar Evaluation Handbook*, Artech House, Dedham, MA, 1991, Ch. 3.

[6] Nathanson, F.E., *Radar Design Principles*, 2d ed., McGraw-Hill, Inc., New York, 1991, Ch. 2.

[7] Stutzman, W.A., and Thiele, G.A., *Antenna Theory and Design*, 2d ed., Wiley, New York, 1997.

[8] Difranco, J.V., and Rubin, W.L., *Radar Detection*, Artech House, Dedham, MA, 1980, Ch. 12.

[9] Sullivan, R.J., *Radar Foundations for Imaging and Advanced Concepts,* SciTech Publishing, Inc., Raleigh, NC, 2004, Ch. 1.

2.20 PROBLEMS

1. Target received power: Using equation (2.8), determine the single-pulse received power level from a target for a radar system having the following characteristics:

Transmitter:	100 kilowatt peak
Frequency:	9.4 GHz
Antenna Gain:	32 dB
Target RCS:	0 dBsm
Target Range:	50 km

2. Using equation (2.10), determine the receiver noise power (in dBm) for a receiver having a noise figure of 2.7 dB and an instantaneous bandwidth of 1 MHz.

3. Using equation (2.11), determine the single-pulse SNR for the target described in problem 1 if the receiver has a noise figure of 2.7 dB and an instantaneous bandwidth of 1 MHz.

4. Ignoring any losses, using equation (2.8), determine the single-pulse received power level (in dBm) for a 1 square meter target at a range of 60 km for radar systems with the following characteristics:

	P_t (watts)	G	Frequency
Radar a	10,000	36 dB	9.4 GHz
Radar b	100,000	31 dB	9.4 GHz
Radar c	100,000	31 dB	2.8 GHz
Radar d	80,000	36 dB	9.4 GHz

5. Using equation (2.11), determine the SNR for the four conditions described in problem 4 for the following noise-related characteristics. Bandwidth for both frequencies is 10 MHz, the noise figure for 9.4 GHz systems is 3.2 dB, and the noise figure for the 2.8 GHz system is 2.7 dB.

6. Using equation (2.17), determine the four answers in problems 4 and 5 for the following loss conditions:

$L_{tx} = 2.1$ dB
$L_{rx} = 4.3$ dB.

7. If atmospheric propagation losses of 0.12 dB per km (two-way) are also considered, determine the resulting SNR values in problem 6.

8. If we desire the SNR in problem 7 to be the same as in problem 4, we can increase the SNR in problem 7 by transmitting, receiving, and processing multiple pulses. Use equation (2.14) to determine how many pulses we have to transmit to recover from the losses added in problems 6 and 7. (Hint: instead of solving the problem from the beginning, merely determine the relationship between the number of pulses transmitted and the SNR improvement.)

9. A radar system provides 18 dB SNR for a target having an RCS of 1 square meter at a range of 50 km. Ignoring the effects of atmospheric propagation loss, using equation (2.18), determine the range at which the SNR be 18 dB if the target RCS is reduced to:

a. 0.5 square meters.
b. 0.1 square meters.

10. A system has a single-pulse SNR of 13 dB for a given target at a given range. Determine the integrated SNR if 20 pulses are coherently processed.

11. A system SNR can be increased by extending the dwell time. If the original dwell time of a system is 1.75 msec, what new dwell time is required to make up for the loss in target RCS from 1 square meter to 0.1 square meters?

12. If the radar system in problem 1 is looking at surface clutter having a reflectivity value of $\sigma^0 = -20$ dB, dBm^2/m^2, if the area of the clutter cell is 400,000 square meters, what is the clutter RCS and the resulting signal-to-clutter ratio (SCR)?

13. If the radar system in problem 1 is looking at volume clutter having a reflectivity value of $\eta = -70$ dBm^2/m^3, if the volume of the clutter cell is 900,000,000 cubic meters, what is the clutter RCS and the resulting SCR?

14. How much power is received by a radar receiver located 100 km from a jammer with the following characteristics? Assume that the radar antenna has an effective area of 1.2 square meters and that the main beam is pointed in the direction of the jammer. Consider only atmospheric attenuation, excluding the effects of, for example, component loss. Provide the answer in terms of watts and dBm (dB relative to a milliwatt.)

Jammer peak power	100 watts
Jammer antenna gain	15 dB
Atmospheric loss	0.04 dB per km (one-way)
Radar average sidelobe level	−30 dB (relative to the main beam)

15. Using the answers from problems 2 and 14 what is the jammer-to-noise ratio (JNR)?

16. If the receiver antenna is not pointed directly at the jammer but a −30 dB sidelobe is, then what would the answer to problem 14 be?

17. What would the resulting JNR be for the sidelobe jamming signal?

18. A search system being designed by an engineering staff has to search the following solid angle volume in the stated amount of time:

Azimuth angle:	90 degrees
Elevation angle:	6 degrees
Full scan time:	1.2 seconds
Maximum range:	50 km
Target RCS:	−10 dBsm

What is the required power aperture product of the system if the system has the following characteristics?

Noise figure:	2.5 dB
System losses:	9.7 dB
Required SNR:	18 dB

19. For the radar system in problem 18, if the antenna has an effective aperture of 0.5 square meters, and the transmit duty cycle is 1%, what is the peak power required?